

Multi-Criteria Partitioning of Multi-Block Structured Grids

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Outline

Background

Algorithms

Tests and Results

Conclusion

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Tests and Results

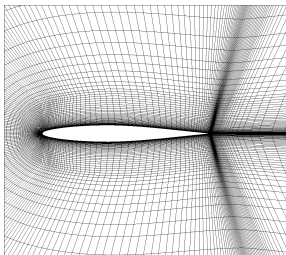
Conclusion

Structured Grid

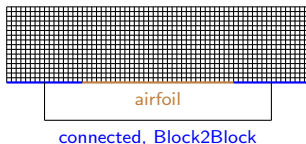
- Structured Grid: Regular connectivity between grid cells.

	$i,j+1$	
$i-1,j$	i,j	$i+1,j$
	$i,j-1$	

- Block: grid unit equivalent to a single rectangle.



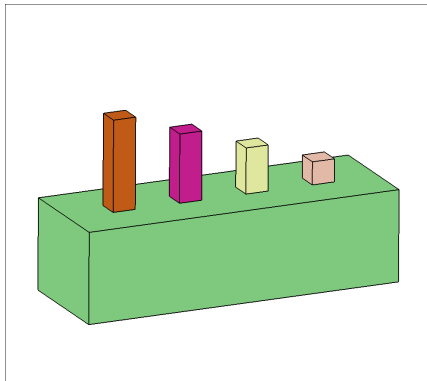
Airfoil Grid



Block

Structured Grid

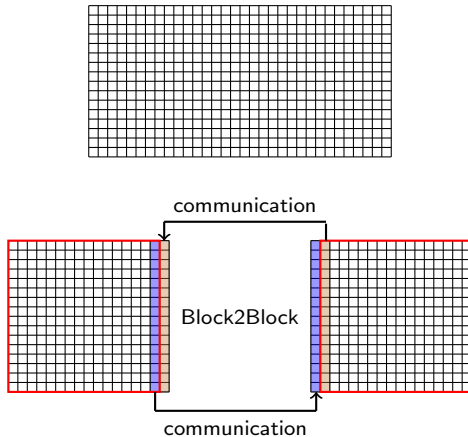
► Multi-Block Structured Grids



Bump3D, 5 blocks

Halo Exchange

Split a block into 2 partitions and assign each partition to a node:

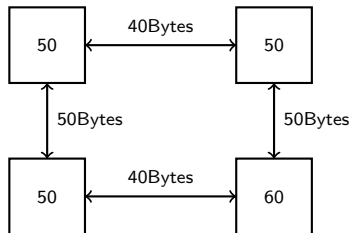


Hybird Programming Model

Hybrid programming model:

- ▶ 1 MPI process per node and spawn threads within a node.
- ▶ Assume shared memory copy takes no time.

Partition 4 blocks onto 2 nodes:



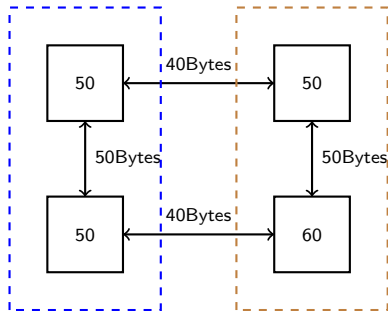
Average Workload $\overline{W}$ 105

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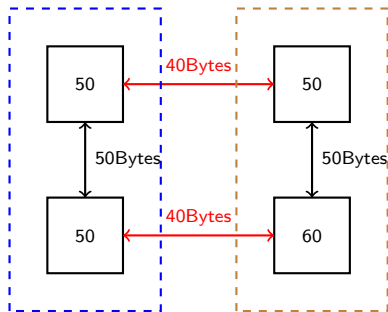
Average Workload $\overline{W}$ 105
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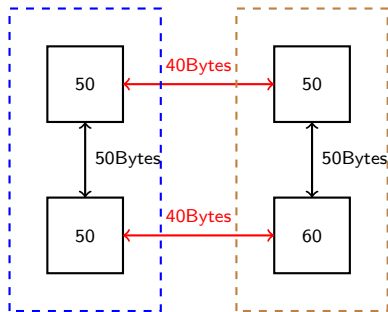
Average Workload $\overline{W}$	105
Imbalance	5/105
Edge Cuts	2
Communcation Volume	80 Bytes

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Edge Cuts	2
Communcation Volume	80 Bytes
Shared Memory Copy	100 Btyes

Objectives

Given the number of partitions n_p , workload per partition $\overline{W}$, the partitioner should:

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- ▶ Achieve load balance
 - Trade off load balance for communication cost
- ▶ Minimize communication cost
 - Reduce the inter-node communication
 - Convert Block2Block communication to shared memory copy

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State-of-the-art Methods

The state-of-the-art methods can be divided into two strategies:

- ▶ Top-down strategy:
 - Cut large blocks and assign sub-blocks to partitions.
 - Group small blocks to fill partitions.

Examples:

Greedy [Ytterström 97]

Recursive Edge Bisection (REB) [Berger 87]

Integer Factorization (IF)

- ▶ Bottom-up strategy:

Transform the problem to graph partitioning and use graph partitioner.

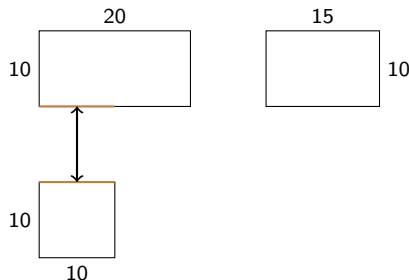
Examples:

Metis [Karypis 94], Scotch [Roman 96], Chaco [Leland 95]

Greedy Algorithm

Greedy Algorithm:

- ▶ Assign (part of) the largest block to the most underload partition.
- ▶ Cut at the longest edge of a block.

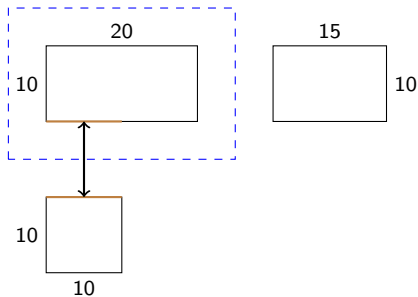


$$\overline{W} = 300 \quad W_p = 0$$

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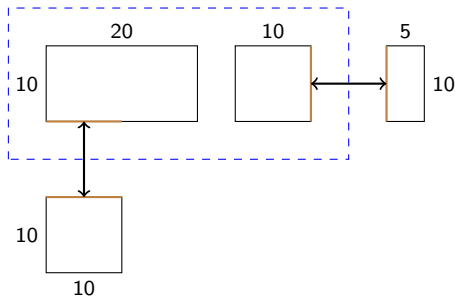


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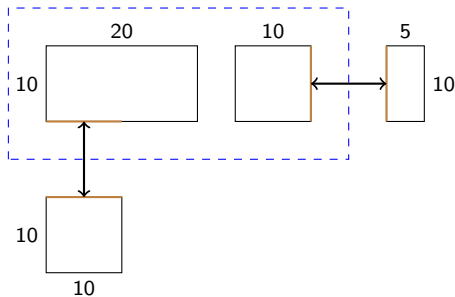


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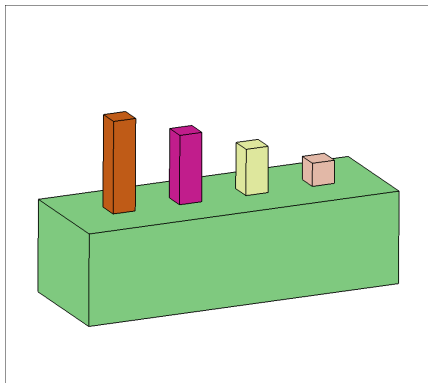


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- ✗ Ignores the connectivity between blocks.
- ✗ Creates excessive small blocks when cutting a large block

Greedy Algorithm

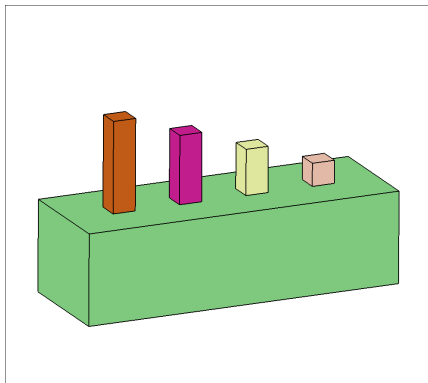
Bump3D grid: 5 blocks, the largest block is 27 times larger than the rest.



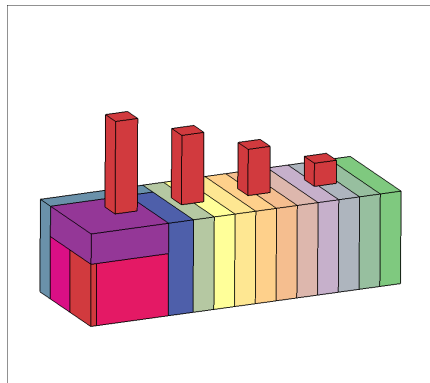
Bump3D blocks

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Bump3D blocks



Greedy, 16 partitions

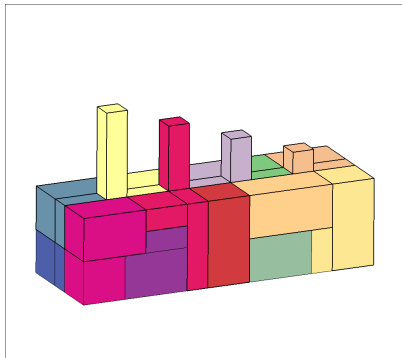
Bottom-up Strategy

Bottom-up: Convert the structured grid partitioning to general graph partitioning. For a graph partitioner to work well, it needs **large number of vertices per partition**.

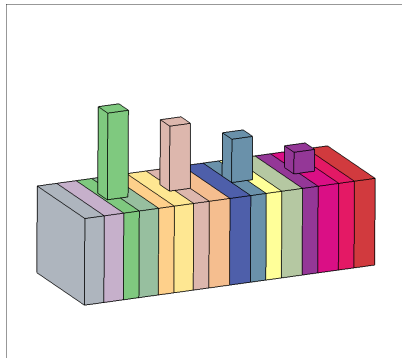
1. **Over-decompose** blocks, construct graph with blocks as vertices
2. Apply graph partitioner: Metis, Scotch, Chaco, etc
3. Merge blocks within one partition

Bottom-up Strategy

Use Metis as the graph partitioner to generate 16 partitions with different over-decomposition method.



Over-Decompose to elementary blocks



Over-Decompose with IF

Limitations of State-of-the-art Methods

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- ▶ Flat MPI, ignore the shared memory on the algorithm level.
- ▶ The communication performance does not distinguish the shared memory copy and inter-nodes data transfer.
- ▶ Primarily focus on reducing communication volume, ignore the effect of network's latency.

Our Partition Algorithms

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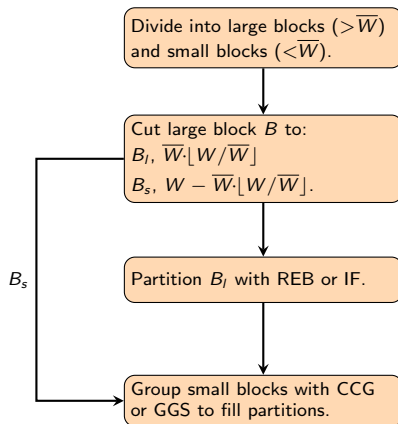
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 - Modify Recursive Edge Bisection (REB) and Integer Factorization (IF) for cutting large blocks ($W > \overline{W}$).
 - Propose Cut-Combine-Greedy (CCG) and Graph-Grow-Sweep (GGS) for grouping small blocks.

Our Partition Algorithms

Cut large blocks $\rightarrow$ Group small blocks:



Measure of Communication cost

$\alpha - \beta$ model: α latency (s), β bandwidth (bytes/s)

$$\text{Cost}(s) = \alpha + \frac{\text{sizeof}(\text{message})}{\beta}$$

For Block2Block message:

$$t_{b2b} = \alpha + \frac{\#Halo \cdot FaceArea \cdot \text{sizeof}(\text{cell})}{\beta}$$

Sum over all Block2Block messages:

$$\sum t_{b2b} = \alpha \cdot \sum \text{Edge Cuts} + \frac{\text{Communication Volume}}{\beta}$$

Cut Large Block: find a cut

Input: Block B , workload W_{cut} , tolerance ϵ , partition P (optional)

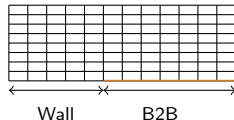
Output: a cut satisfies:

- (1) the cut-off sub-block fits in $[W_{cut}(1 - \epsilon), W_{cut}(1 + \epsilon)]$
- (2) introduces minimum communication cost δt

Find a cut

```

1: for  $i = x, y, z$  do
2:   Get area of  $i$ 's norm face  $A_i$ 
3:    $f = \text{floor}(W_{cut}(1 - \epsilon)/A_i)$ 
4:    $c = \text{ceil}(W_{cut}(1 + \epsilon)/A_i)$ 
5:   for  $\text{pos} \in [\text{posFloor}, \text{posCeil}]$  do
6:      $\delta t_{cut} = \sum_{b2b \text{ cut}} \alpha + t_{b2b}(A_i) - \sum_{B_j \in P} t_{b2b}(\text{cut}, B_j)$ 
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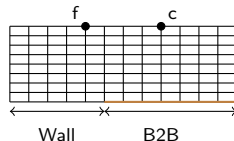
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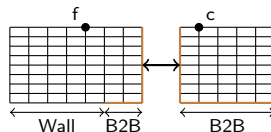
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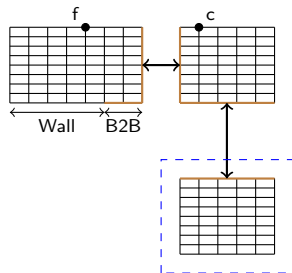
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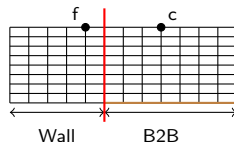
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Cut Large Block: REB

Recursive Edge Bisection (REB):

Algorithm: REB

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1: function REB_BLOCK( $B, n_p$ )
   ▷ Block  $B$  fits in  $n_p$  partitions.
2: if  $n_p == 1$  then
3:   return
4:    $W = B$ 's workload
5:    $W_l = W \cdot \frac{\lfloor n_p/2 \rfloor}{n_p}, W_r = W - W_l$ 
6:   find_min_cut( $B, W_l, \epsilon, \text{cut}$ )
7:   cut  $B$  into  $B_l$  of  $W_l$  and  $B_r$  of  $W_r$ 
8:   reb_block( $B_l, \lfloor n_p/2 \rfloor$ )
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$$n_p = 7$$

7



Cut Large Block: REB

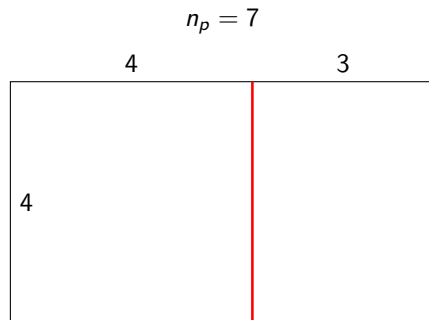
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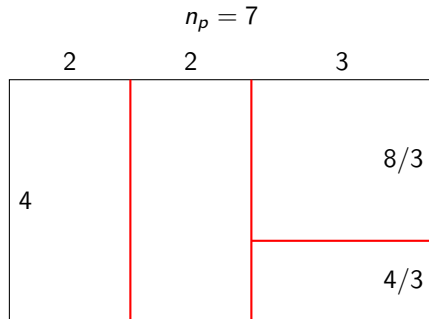
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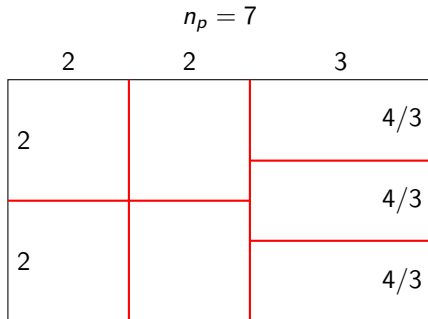
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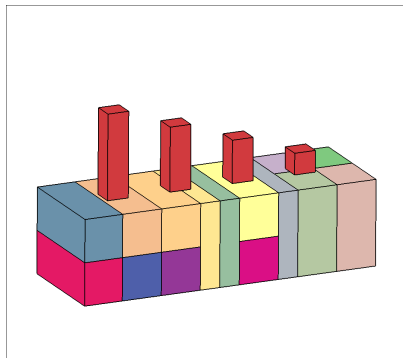
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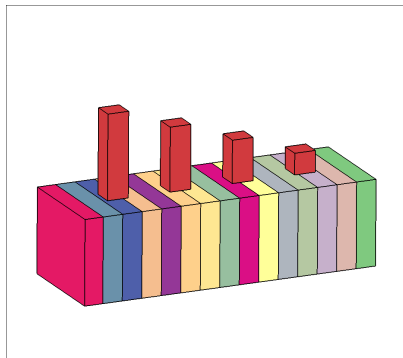


Cut Large Block: REB

Using REB to split Bump3D into 16 partitions with different α , β values:



$$\alpha = 10^{-5}, \beta = 10^9$$



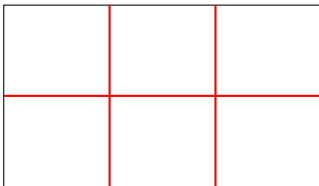
$$\alpha = 10^{-4}, \beta = 10^9$$

Cut Large Block: IF

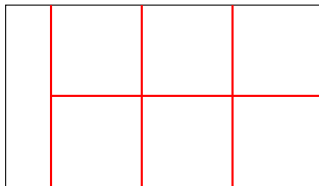
Integer Factorization (IF):

$$n_p = n_x \cdot n_y \cdot n_z, \quad \frac{n_x}{l_x} \approx \frac{n_y}{l_y} \approx \frac{n_z}{l_z}$$

If n_p is prime, cut off one partition and factorize the rest.



$$l_x = 7, l_y = 4, n_p = 6$$

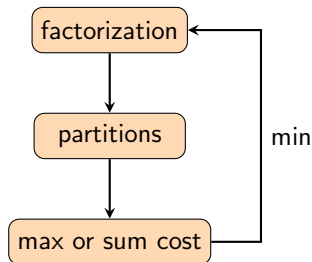


$$l_x = 7, l_y = 4, n_p = 7$$

Cut Large Block: IF

Generalize IF by using $\alpha - \beta$ cost function:

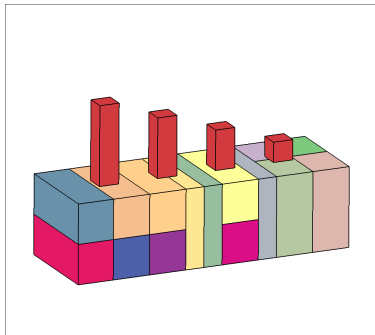
- ▶ Compare $n_p = n_x n_y n_z$ and $n_p = 1 + n_x n_y n_z$ for every case.
- ▶ Choose the factorization based on max or sum $\alpha - \beta$ cost.



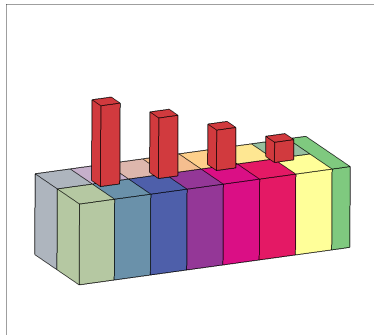
Cut Large Block: Compare REB and IF

Compare REB and IF on Bump3D, $\alpha = 10^{-5}$, $\beta = 10^9$, $n_p = 16$.

In general, IF is better at reducing edge cuts and REB better at reducing communication volume.



REB, edge cuts 66, communication volume 1.57×10^6

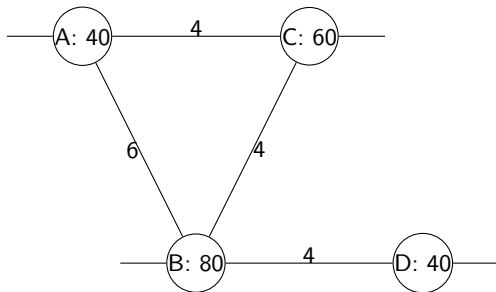


IF, edge cuts 66, communication volume 1.61×10^6

Group Small Blocks: CCG

Cut-Combine-Greedy (CCG): cut and combine small blocks in a greedy fashion.

- ▶ Include (part of) the block reducing max communication cost to the partition.
- ▶ Convert Block2Block communication to shared memory copy.

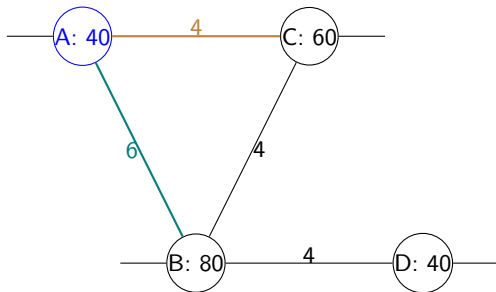


$$\overline{W} = 160 \quad W_p = 0$$

Group Small Blocks: CCG

Cut-Combine-Greedy (CCG): cut and combine small blocks in a greedy fashion.

- ▶ Include (part of) the block reducing max communication cost to the partition.
- ▶ Convert Block2Block communication to shared memory copy.



$$\overline{W} = 160 \quad W_p = 40$$

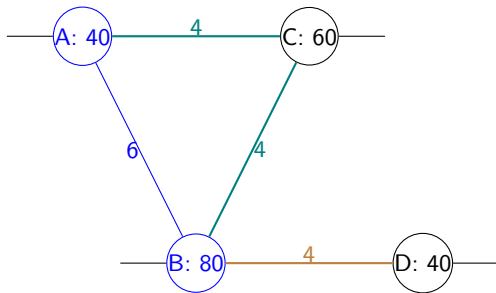
B cost: -6

C cost: -4

Group Small Blocks: CCG

Cut-Combine-Greedy (CCG): cut and combine small blocks in a greedy fashion.

- ▶ Include (part of) the block reducing max communication cost to the partition.
- ▶ Convert Block2Block communication to shared memory copy.



$$\overline{W} = 160 \quad W_p = 120$$

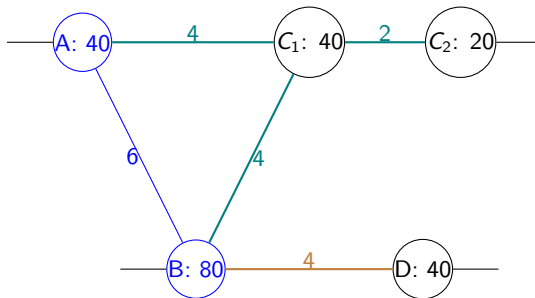
C overload

D cost: -4

Group Small Blocks: CCG

Cut-Combine-Greedy (CCG): cut and combine small blocks in a greedy fashion.

- ▶ Include (part of) the block reducing max communication cost to the partition.
- ▶ Convert Block2Block communication to shared memory copy.



$$\overline{W} = 160 \quad W_p = 120$$

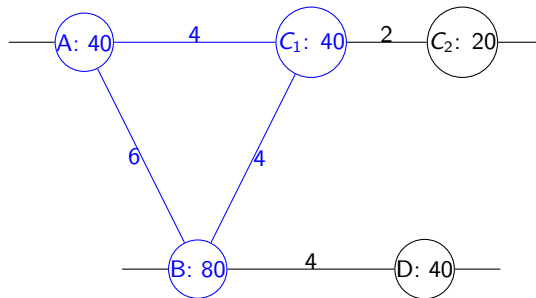
$$C_1 \text{ cost: } -4-4+2 = -6$$

$$D \text{ cost: } -4$$

Group Small Blocks: CCG

Cut-Combine-Greedy (CCG): cut and combine small blocks in a greedy fashion.

- ▶ Include (part of) the block reducing max communication cost to the partition.
- ▶ Convert Block2Block communication to shared memory copy.



$$\overline{W} = 160 \quad W_p = 160$$

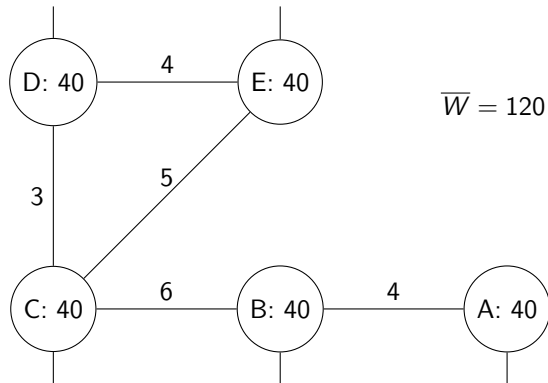
Convert cost: 14

Introduce new cost: 2

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.

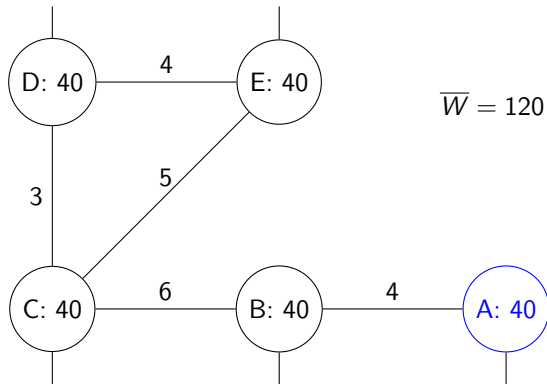


$$\overline{W} = 120 \quad W_1 = 0 \quad W_2 = 0$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.

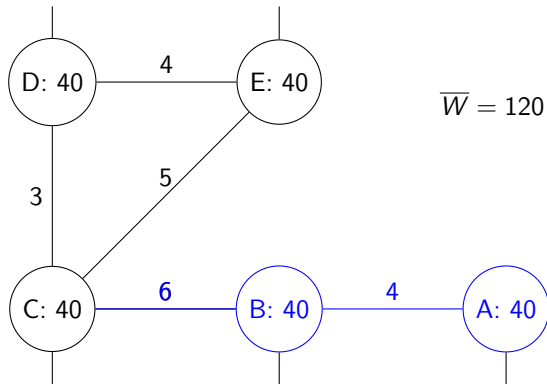


$$\overline{W} = 120 \quad W_1 = 40 \quad W_2 = 0$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.

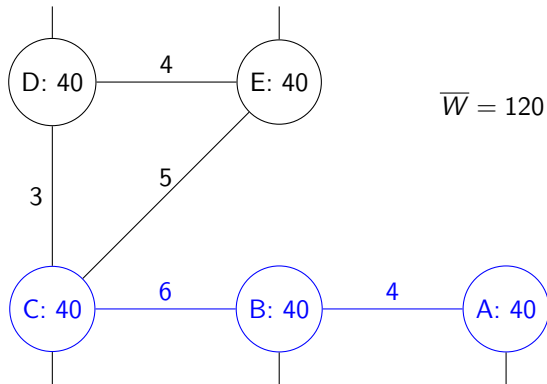


$$\overline{W} = 120 \quad W_1 = 80 \quad W_2 = 0$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.

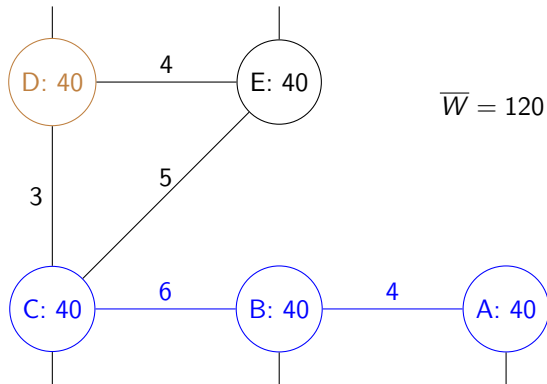


$$\overline{W} = 120 \quad W_1 = 120 \quad W_2 = 0$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
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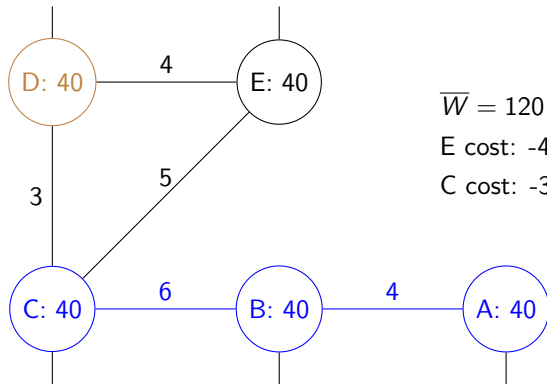


$$\overline{W} = 120 \quad W_1 = 120 \quad W_2 = 40$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.



$$\overline{W} = 120 \quad W_1 = 120 \quad W_2 = 40$$

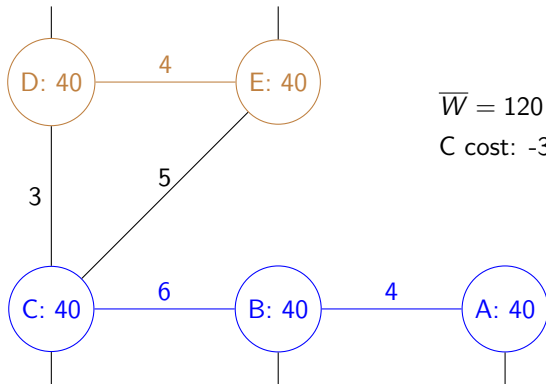
E cost: -4

C cost: $-3 + 6 = 3$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.



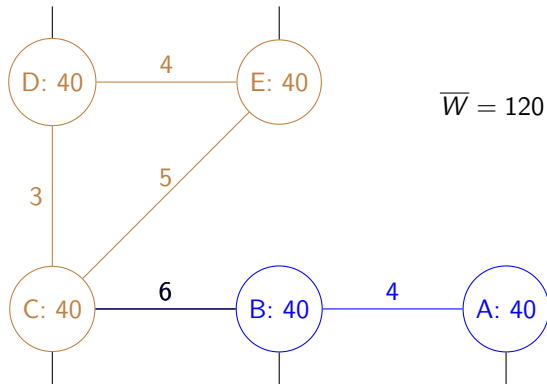
$$\overline{W} = 120 \quad W_1 = 120 \quad W_2 = 80$$

$$C \text{ cost: } -3 - 5 + 6 = -2$$

Group Small Blocks: GGS

Graph-Growth-Sweep (GGS): Repeatedly use graph growing to group small blocks.

- ▶ Convert Block2Block communication to shared memory copy.
- ▶ Avoid cutting blocks.



$$\overline{W} = 120 \quad W_1 = 80 \quad W_2 = 120$$

Group Small Blocks: Compare CCG and GGS

To compare CCG and GGS:

When the #blocks is large, CCG

- ✓ converts more communication to shared memory copy.
- ✗ creates more cuts and new Block2Block communications.

GGS

- ✗ converts less communication to shared memory copy.
- ✓ avoids cutting blocks and introduces less new Block2Block communications.

Outline

Background

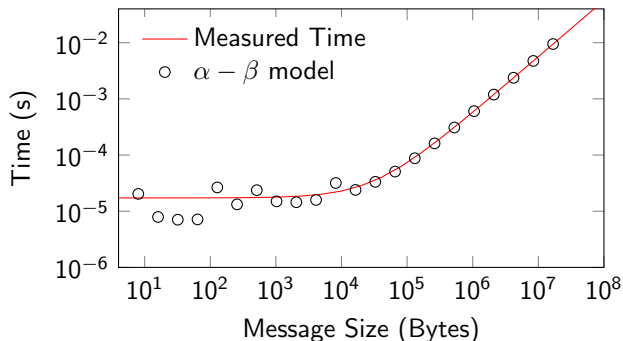
Algorithms

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Hardware and Network Specs

- ▶ Our experiments are performed on Mira.
IBM BlueGene/Q cluster, 16 cores per node
- ▶ The latency $\alpha = 1.73 \times 10^{-5} \text{s}$ and bandwidth $\beta = 1.77 \times 10^9 \text{ bytes/s}$ are measured by a pingpong test.



Numerical Experiment

Implement a Jacobi type solver with MPI and OpenMP.

- ▶ Assign each MPI process to a node and spawn one OpenMP thread per core.
- ▶ Master thread calls MPI non-blocking routines which overlaps with shared memory copy.

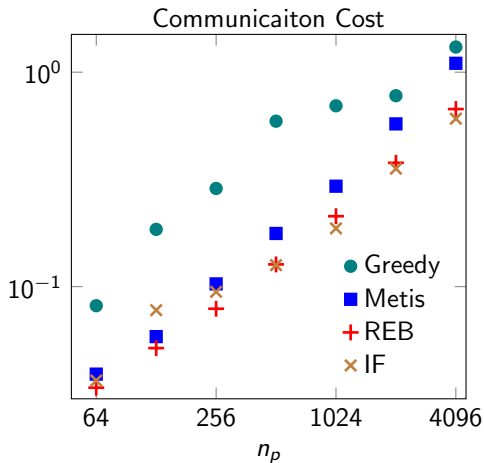
Experimental Jacobi Solver

- 1: **for** $i = 1 \rightarrow \text{NSTEP}$ **do**
 - ▷ `#pragma omp for`
 - 2: Copy halo data to sending buffer
 - ▷ `#pragma omp barrier`
 - ▷ `#pragma omp master`
 - 3: Update halo using non-blocking p2p communication
 - ▷ `#pragma omp for`
 - 4: Copy halo data via shared memory within node
 - ▷ `#pragma omp barrier`
 - ▷ `#pragma omp for`
 - 5: Copy data from receiving buffer to halo region
 - ▷ `#pragma omp barrier`
 - ▷ `split blocks evenly among threads`
 - 6: Computation
 - ▷ `#pragma omp barrier`
-

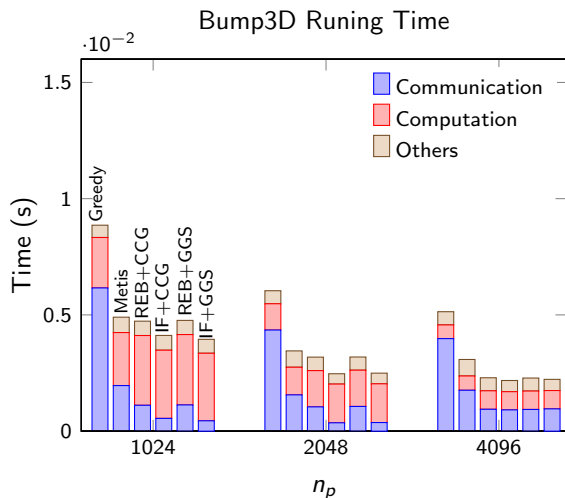
Bump3D Metrics: Communication Cost

Refine Bump3D 4 times in each direction, 8.3×10^7 cells in total.

Beyond 512 partitions: Greedy $>$ Metis $>$ REB $\approx$ IF

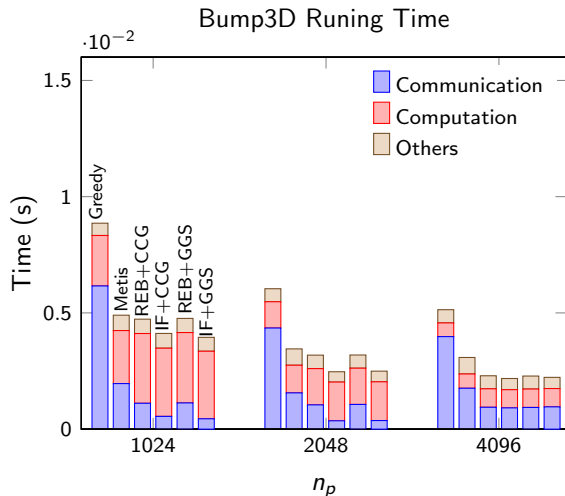


Bump3D Running Time



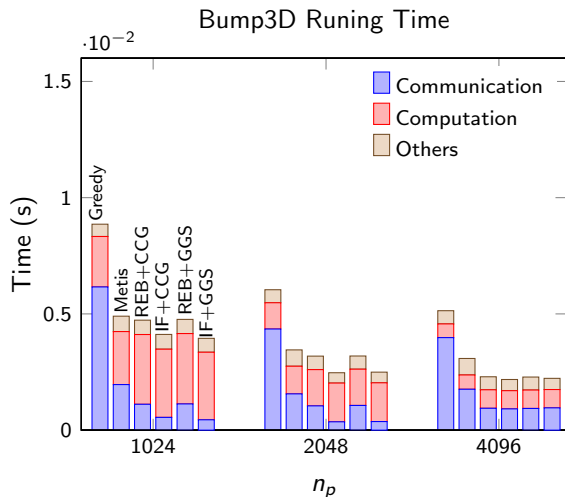
► Consistent with $\alpha - \beta$ cost.

Bump3D Running Time



- Consistent with $\alpha - \beta$ cost.
- Latency has more effect.

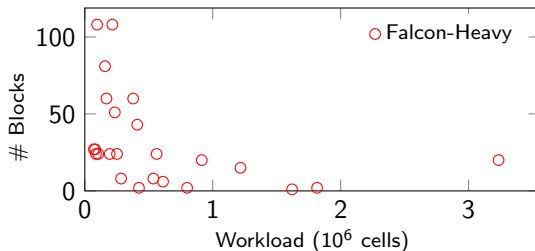
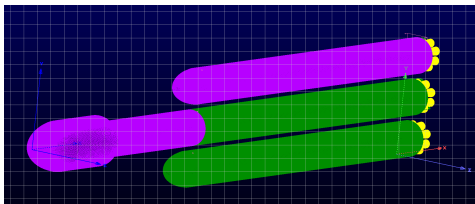
Bump3D Running Time



- Consistent with $\alpha - \beta$ cost.
- Latency has more effect.
- At 4096 partitions, IF achieves 5.80x over Greedy and 2.56x over Metis in communication.

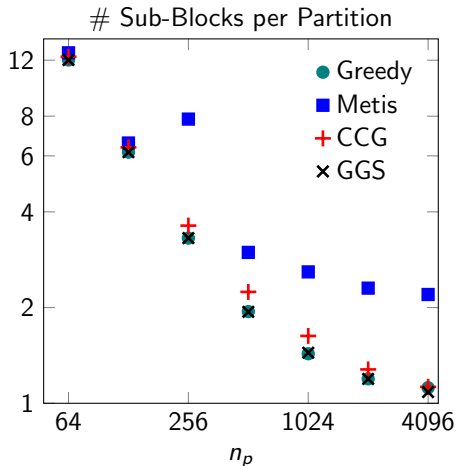
Space X's Falcon-Heavy Grid

The Space X's Falcon-Heavy grid and its blocks' distribution:



Falcon-Heavy Metrics: # Sub-Blocks

Choose REB+GGG and IF+CCG to present GGS and CCG respectively.



Sub-Blocks:

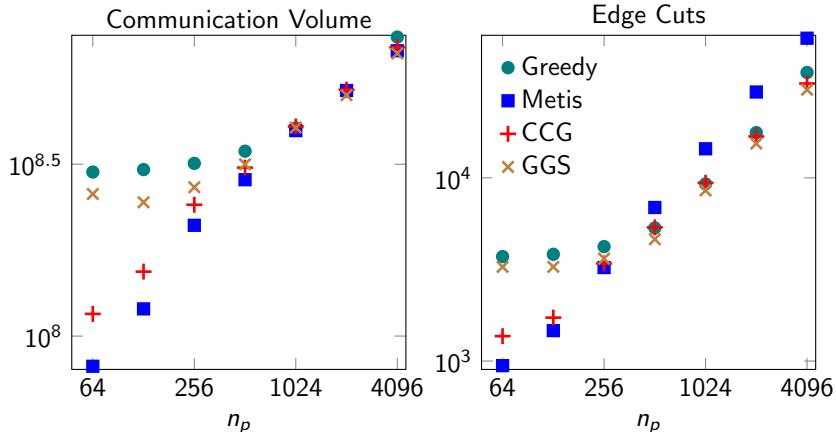
Metis > CCG > Greedy $\geq$ GGS

Dominating Pattern:

- ▶ 64-256 partitions
group small blocks
- ▶ 1024-4096 partitions
both

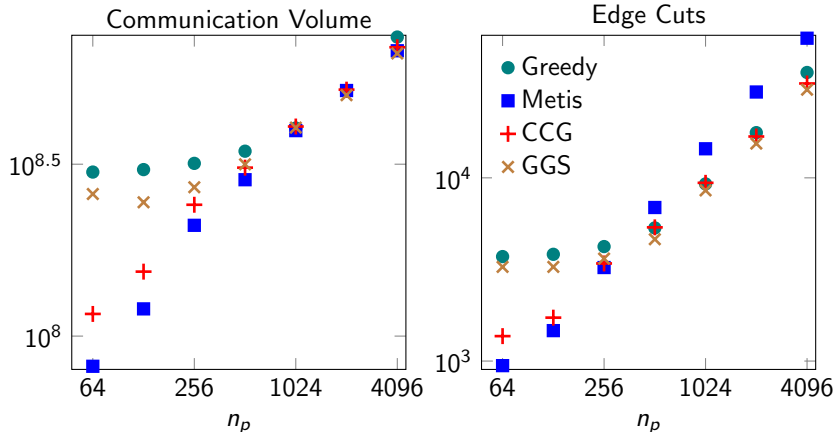
Falcon-Heavy Metrics: Volume and Edge Cuts

- Greedy produces the max communication volume and edge cuts for 64-256 partitions



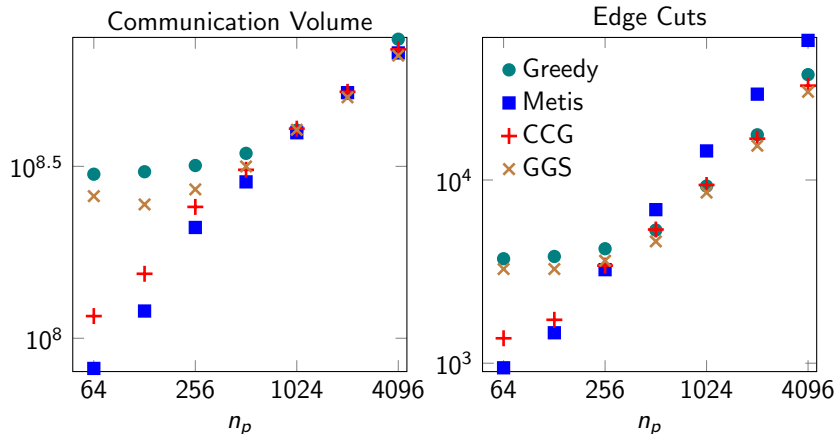
Falcon-Heavy Metrics: Volume and Edge Cuts

- Metis produces the min communication volume and edge cuts for 64-256 partitions, CCG 2nd.

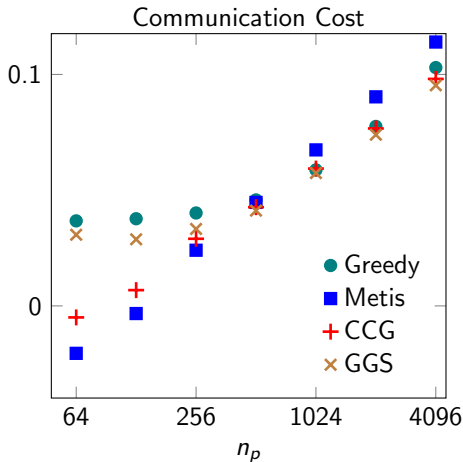


Falcon-Heavy Metrics: Volume and Edge Cuts

- Greedy, CCG, GGS produce close results for 1024-4096 partitions; GGS min.



Falcon-Heavy Metrics: Communication Cost

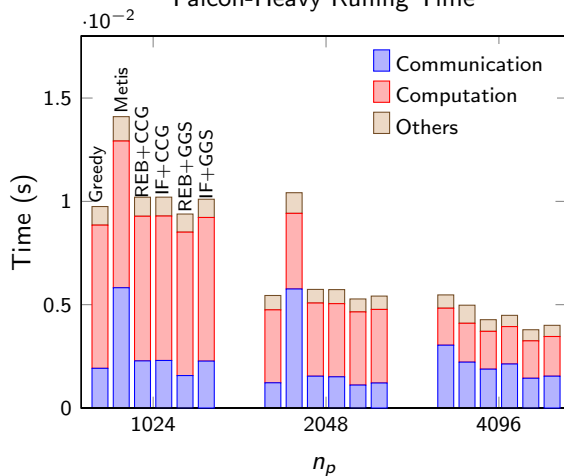


Consistent Pattern with metrics:

- ▶ 64-256 partitions
Metis best, CCG 2nd
- ▶ 1024-4096 partitions
Metis worst, GGS best

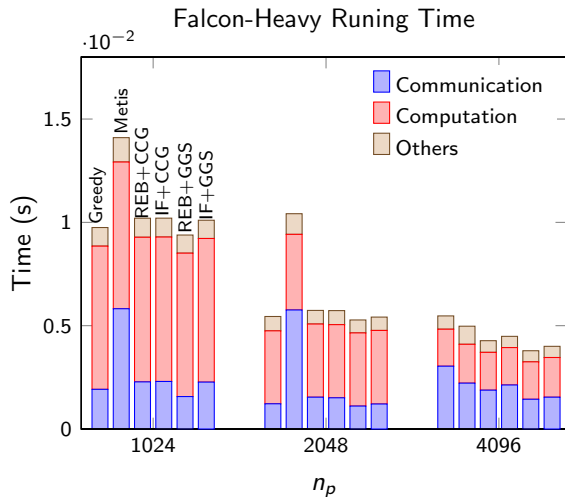
Falcon-Heavy Running Time

Falcon-Heavy Running Time



- Metis produces good result at 4096 partitions

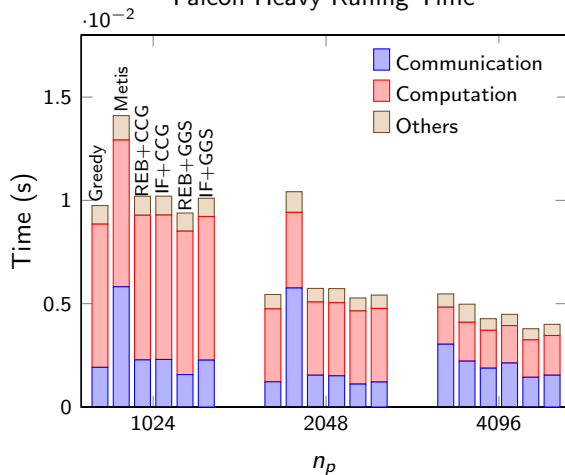
Falcon-Heavy Running Time



- ▶ Metis produces good result at 4096 partitions
- ▶ Greedy creates good result at 1024, 2048 partitions

Falcon-Heavy Running Time

Falcon-Heavy Running Time



- ▶ Metis produces good result at 4096 partitions
- ▶ Greedy creates good result at 1024, 2048 partitions
- ▶ At 4096 partitions, REB+GGS achieves 2.11x over Greedy and 1.54x over Metis in communication.

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- ▶ Use the $\alpha - \beta$ model to construct a cost function incorporating the edge cuts, communication volume and network specifics.

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- ▶ Test our partitioner with a hybrid MPI+OpenMP based Jacobi solver on up to 4096 nodes.

Conclusion

- ▶ Use the $\alpha - \beta$ model to construct a cost function incorporating the edge cuts, communication volume and network specifics.
- ▶ Propose modified REB, IF for cutting large blocks and CCG, GGS for grouping small blocks.
- ▶ Test our partitioner with a hybrid MPI+OpenMP based Jacobi solver on up to 4096 nodes.
- ▶ Achieve significant speedup in communication on both Bump3D and Falcon-Heavy.