

# Portal

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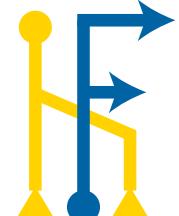
A High-Performance Language and  
Compiler for Parallel N-body Problems

Laleh Aghababaie Beni, Saikiran Ramanan

Prof. Aparna Chandramowlishwaran

University of California, Irvine

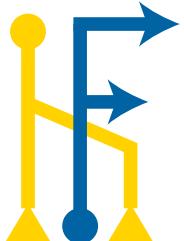
IPDPS 2019





# N-body Problems

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# N-body Problems

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- General representation

$$op_1, \dots, op_m \ K(x_1, \dots, x_m)$$

- Nearest Neighbor

$$\forall q \in Q, \ \arg \min_{r \in R} \|x_q - x_r\|$$





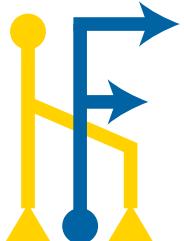
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# N-body Problems

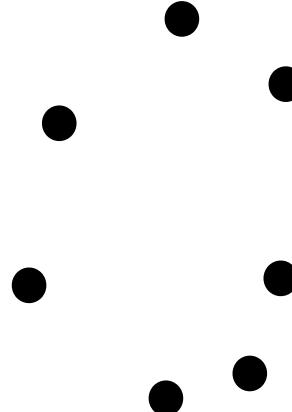
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Nearest Neighbor?



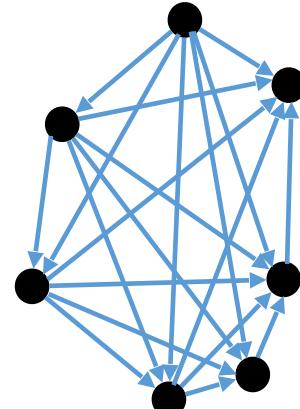


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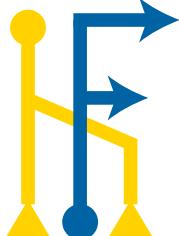
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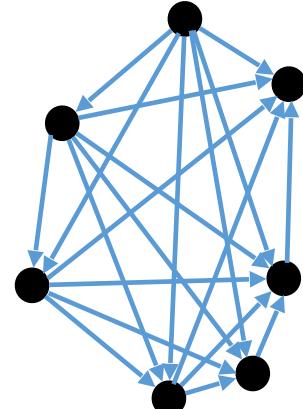
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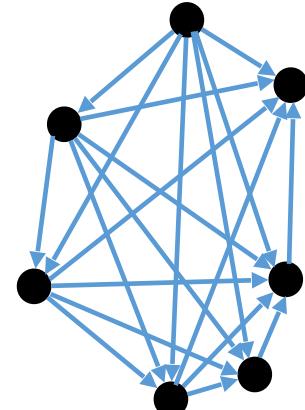
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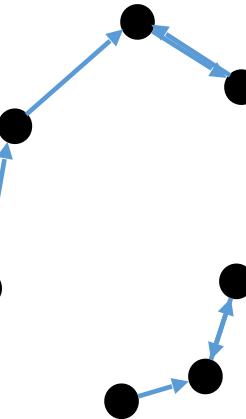
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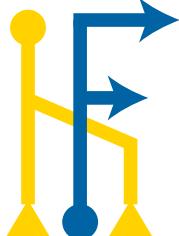
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Nearest Neighbors





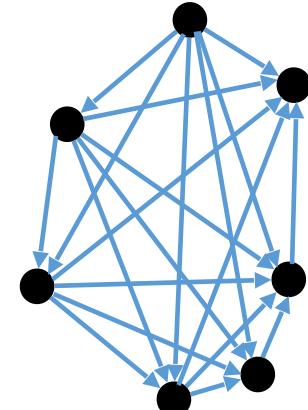
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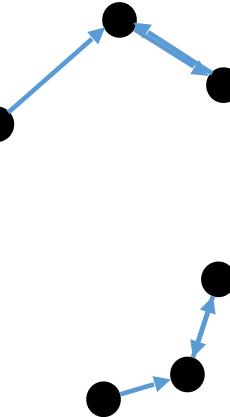
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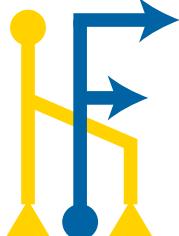
Complexity?



Nearest Neighbor?



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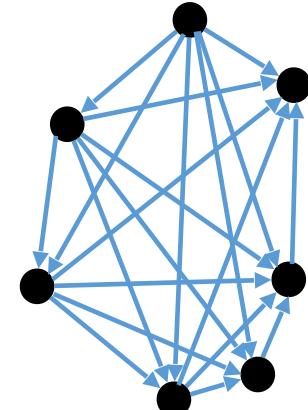
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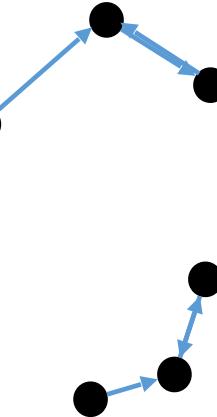
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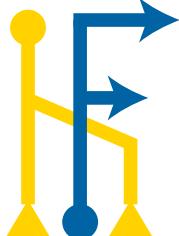
$O(N^2)$



Nearest Neighbor?



Nearest Neighbors





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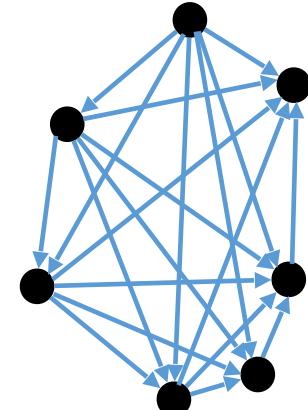
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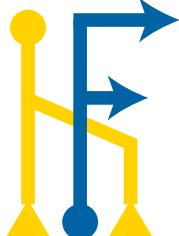
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Nearest Neighbor?



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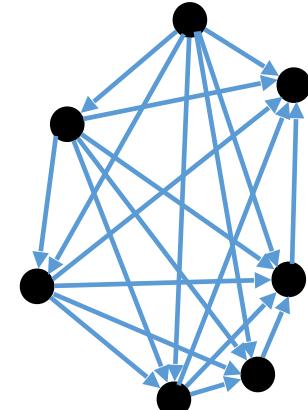
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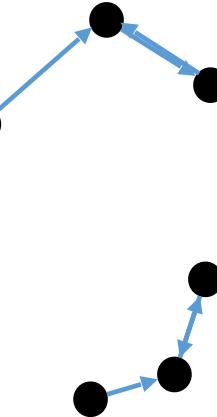
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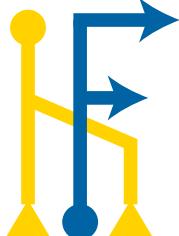
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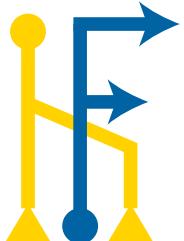


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# N-body Problems

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$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$





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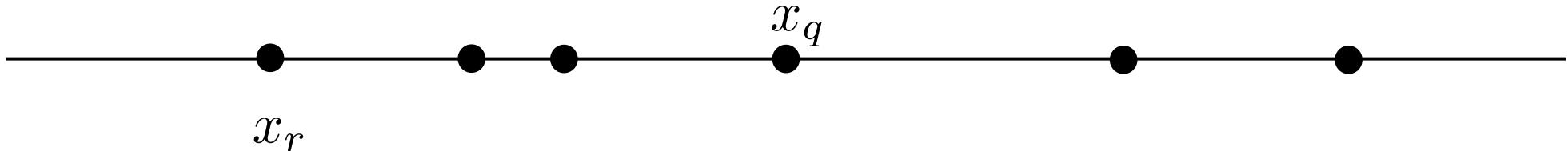
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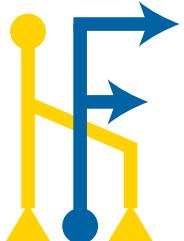
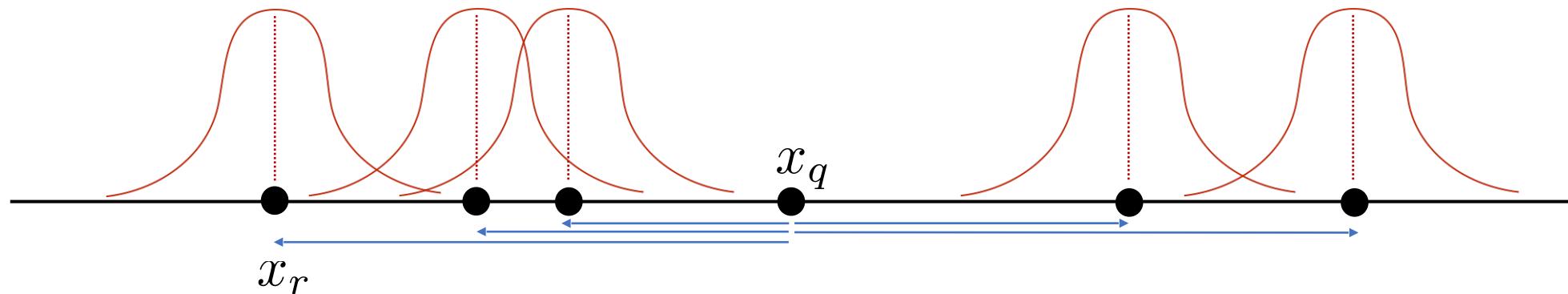
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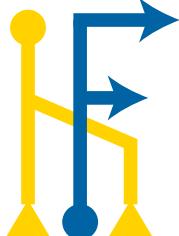
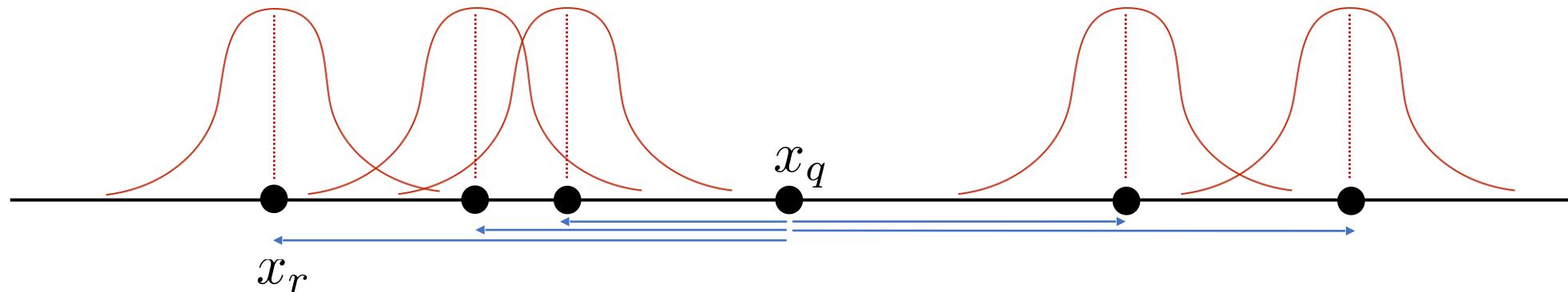
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Complexity?





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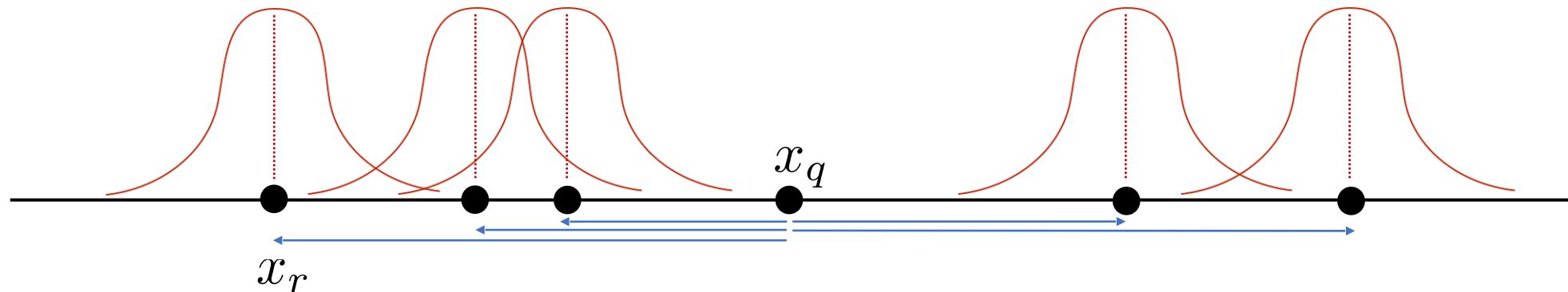
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## Complexity?



$O(N^2)$





# N-body Problems

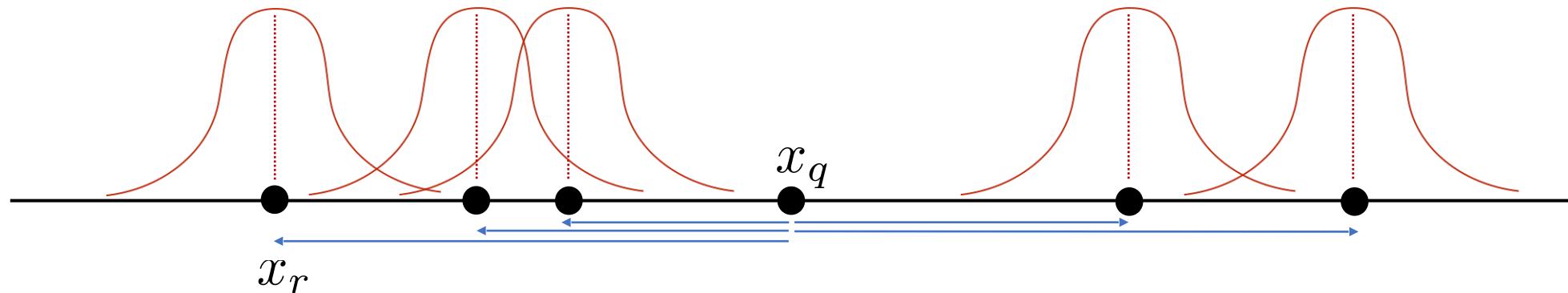
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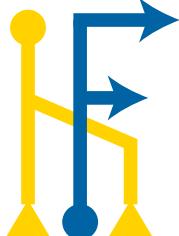
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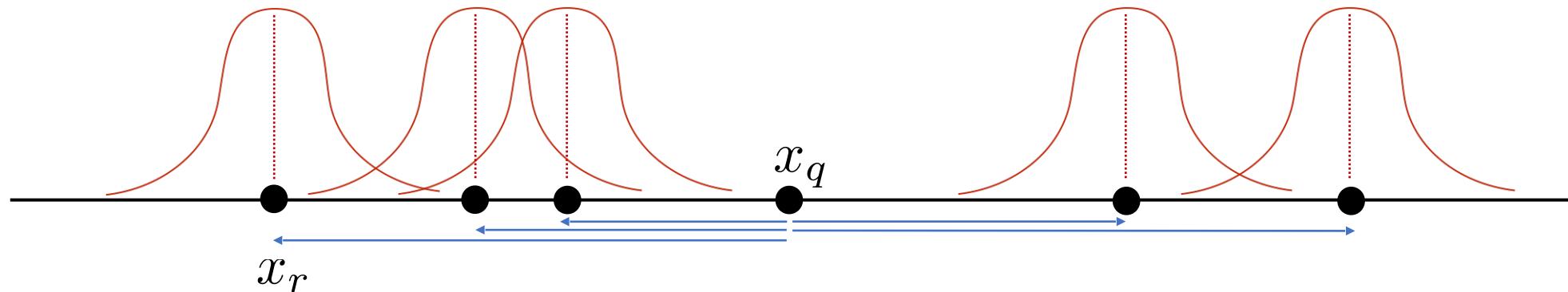
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# Complexity Reduction

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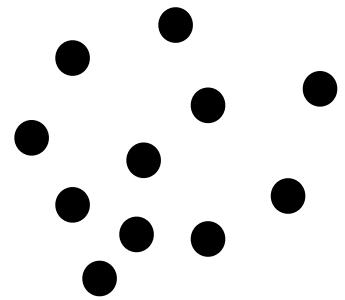




# Complexity Reduction

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- Barnes-Hut [Nature 1986] for force calculation





# Complexity Reduction

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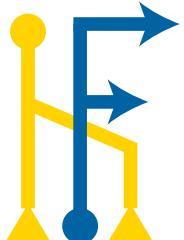
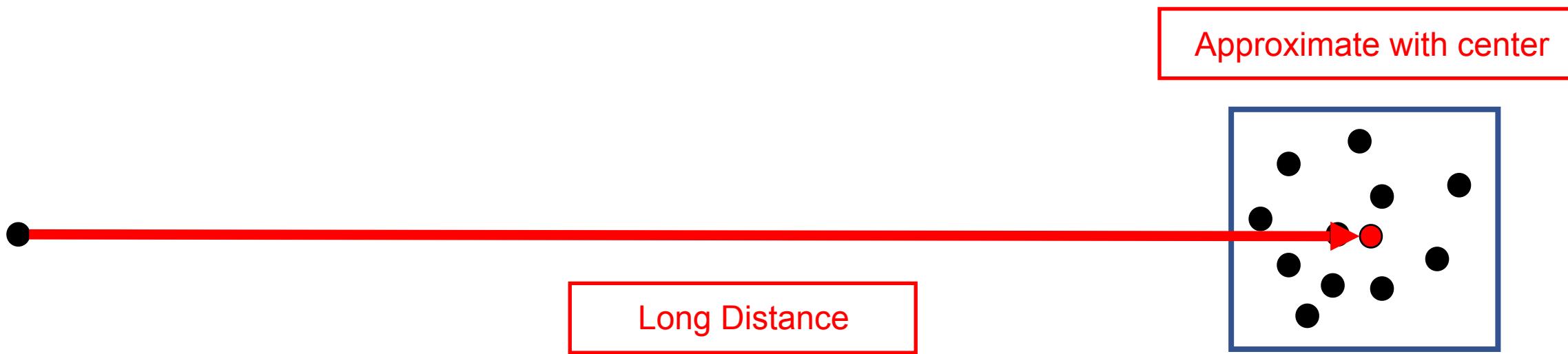




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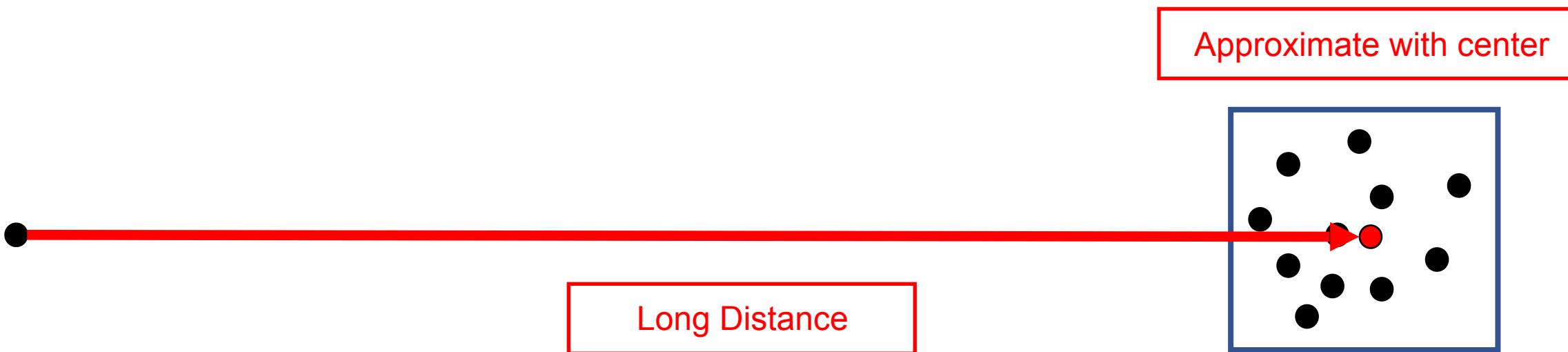
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# Complexity Reduction

- Barnes-Hut [Nature 1986] for force calculation



Asymptotically reduces the complexity:  $O(N^2) \rightarrow O(N \log N)$





# N-body Problems

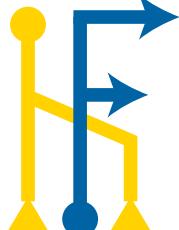
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# N-body Problems

Problem	Operators	Kernel function
All Nearest Neighbor	$\forall, \arg \min$	$\ x_q - x_r\ $
All Range Search	$\forall, \cup \arg$	$I(h_{\min} < \ x_q - x_r\  < h_{\max})$
All Range Count	$\forall, \Sigma$	$I(h_{\min} < \ x_q - x_r\  < h_{\max})$
Naïve Bayes Classifier	$\forall, \arg \max$	$(1/\sqrt{2\pi \Sigma_k })e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)} P(C_k)$
Mixture Model E-step	$\forall, \forall$	$(1/\sqrt{2\pi \Sigma_k })e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)}$
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Kernel Density Estimation	$\forall, \Sigma$	$\phi(\frac{\ x_q - x_r\ }{h})$
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2-Point Correlation	$\Sigma, \Sigma$	$I(\ x_q - x_r\  < h)$
Nadaraya-Watson Regression	$\forall, \Sigma$	$y_r \phi(\frac{\ x_q - x_r\ }{h})$
Thermodynamic Average	$\Sigma, \Sigma$	$\phi(\ x_q - x_r\ )$
Largest-Span Set	max, ..., max	$\Sigma(\ x_q - x_r\ )$
Closest Pair	max, ..., max	$\ x_q - x_r\ $
Minimum Spanning Tree	$\forall, \arg \min$	$\ x_q - x_r\ $
Coulombic Interaction	$\forall, \Sigma$	$\frac{\alpha_q \alpha_r}{\ x_q - x_r\ }$
Average Density	$\Sigma, \Sigma$	$I(\ x_q - x_r\  < h)$
Wave Function	$\forall, \Pi$	$\phi(\ x_q - x_r\ )$
Hausdorff Distance	max, min	$\ x_q - x_r\ $
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Operators → decomposability property over the dataset

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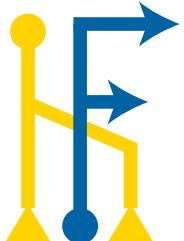
Kernel function → decrease monotonically with distance





# Why N-body Problems?

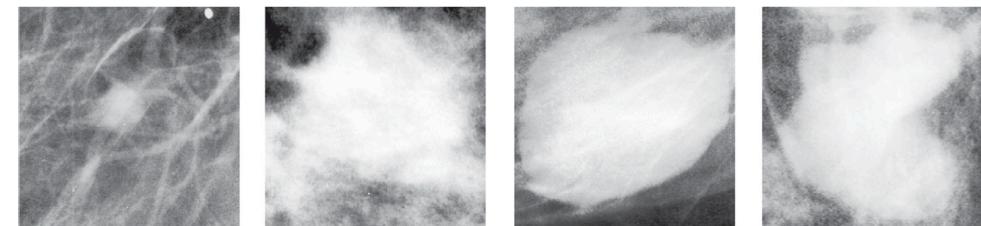
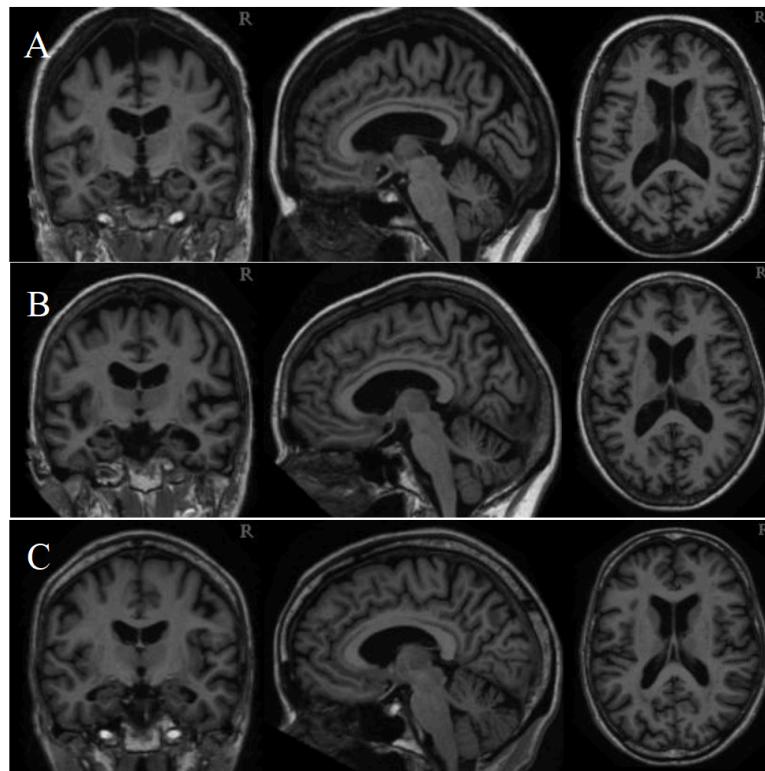
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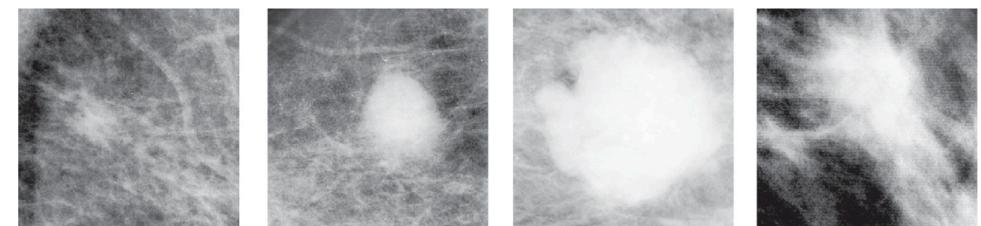


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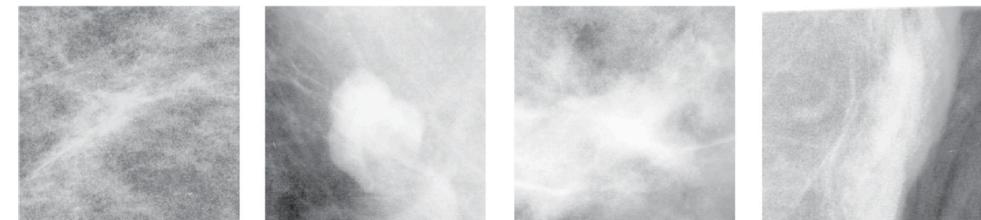
- K-Nearest Neighbors → Alzheimer disease [IJISAE 2016] & breast cancer diagnosis [CBM 2015]



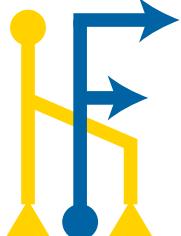
Benign lesion



Malignant lesion



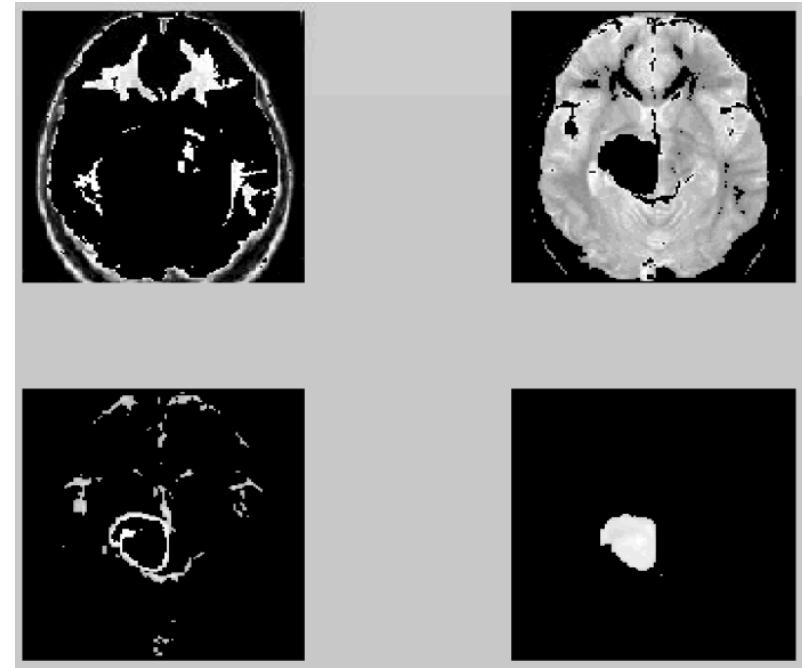
Normal tissue



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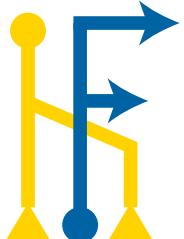
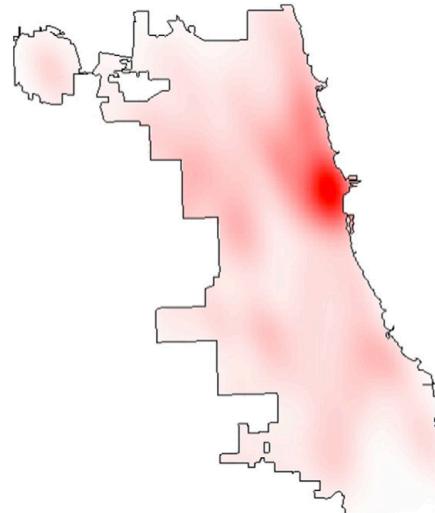




# Why N-body Problems?

---

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# Why N-body Problems?

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- **Applications:** machine learning, computer vision, computational geometry, database, scientific computing, etc.





# Related Work

---





# Related Work

---

- N-body algorithms
  - Physics
    - Barnes-Hut →  $O(N \log N)$ , FMM →  $O(N)$
  - Machine Learning
    - MLPACK [JMLR 2014]
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- Domain-specific language (DSL) and compilers
  - Embedded
    - OptiML → machine learning
    - DeepDSL → deep learning
    - Halide → image processing
  - Stand-alone
    - SCOPE → data analysis





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- Lack efficient optimal algorithms  
- Lack parallelism and scalability





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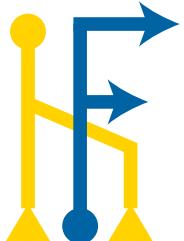
DSLs provide terse and extensible programs while achieving state-of-the-art performance





# Portal

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# Portal

---

- Optimal, parallel, and scalable solution for N-body problems

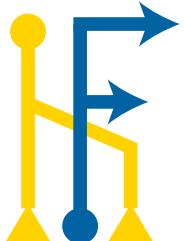




# Portal

---

- Optimal, parallel, and scalable solution for N-body problems
- Embedded language in C++

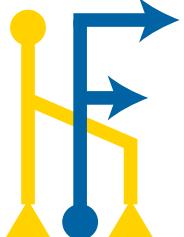




# Portal

---

- Optimal, parallel, and scalable solution for N-body problems
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- Built on top of the *algorithmic framework PASCAL* [Euro-Par 2017]



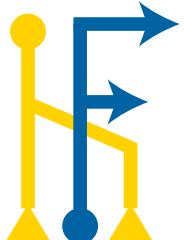


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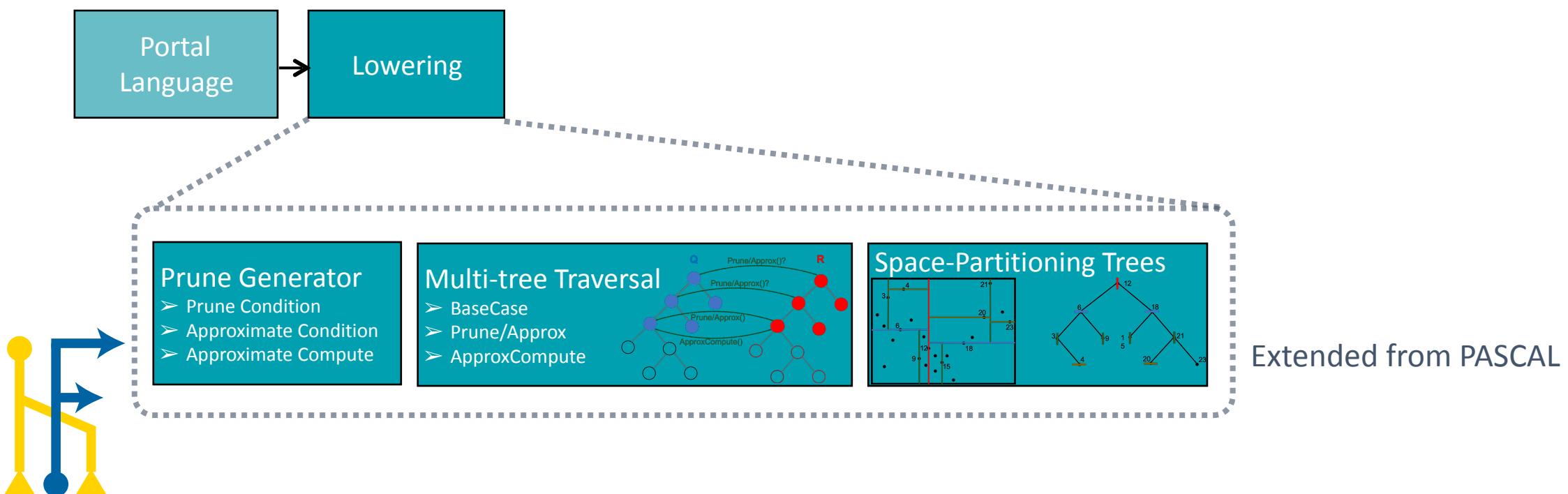
Portal  
Language





# Portal

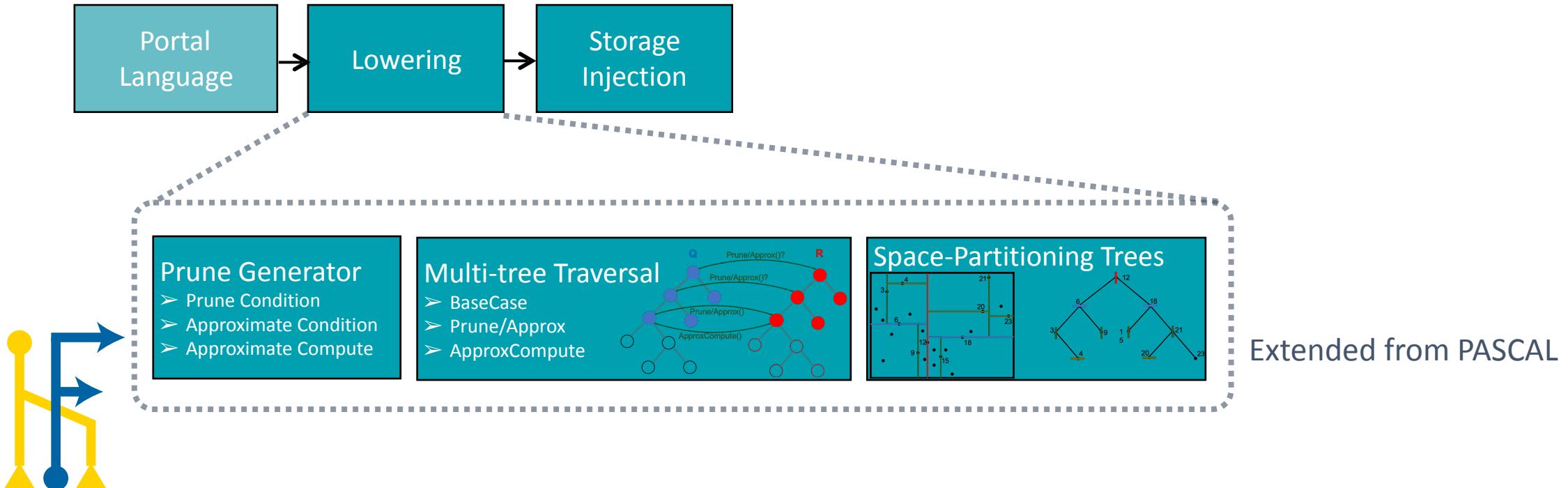
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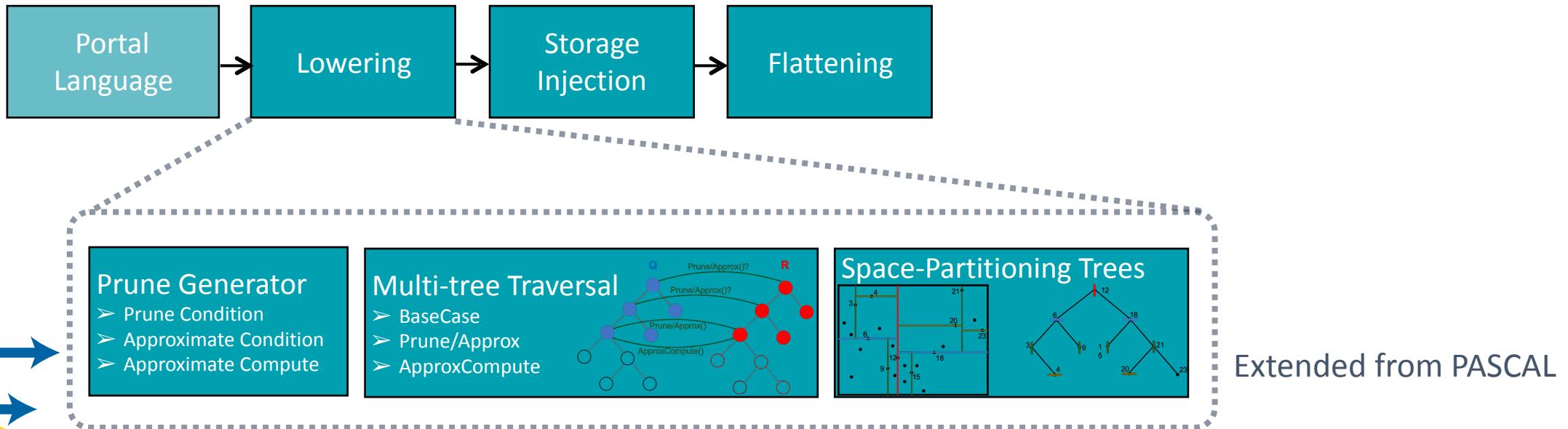
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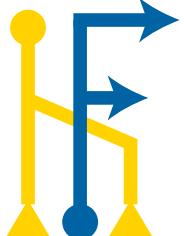
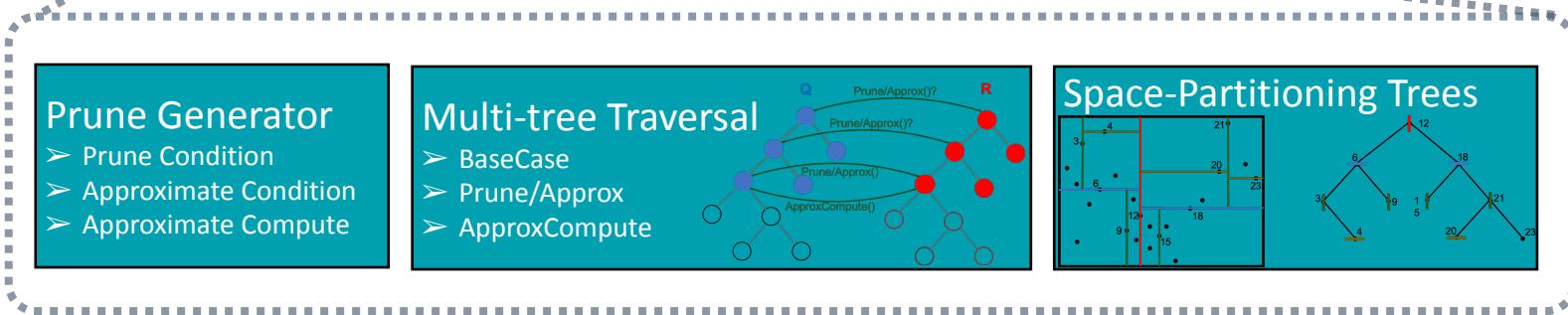
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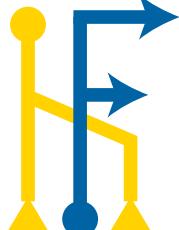
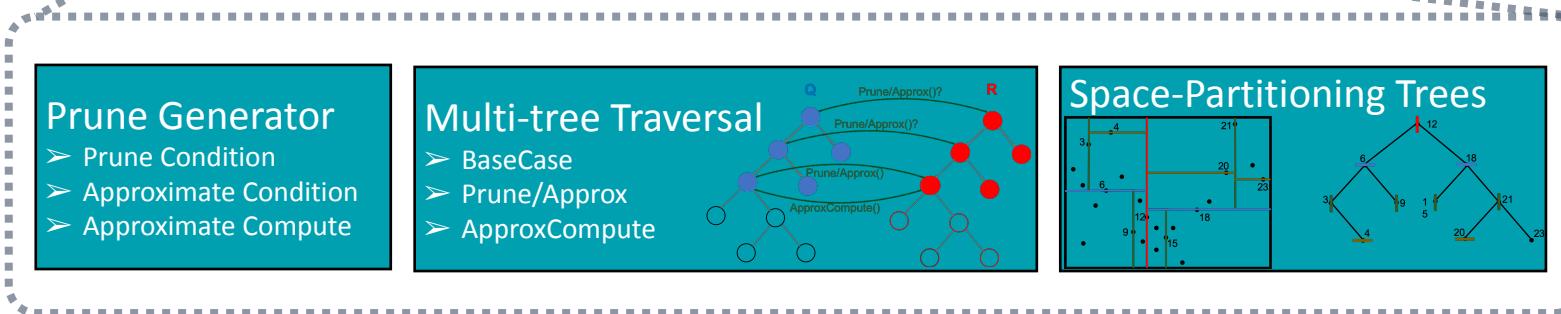
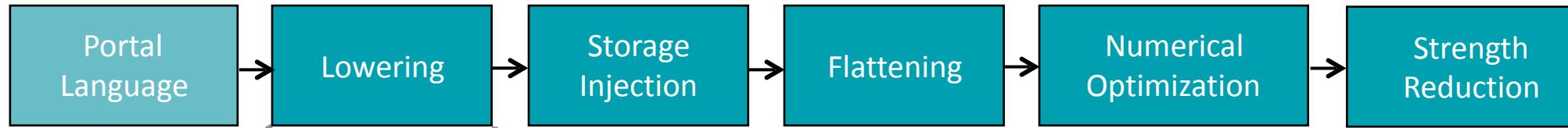
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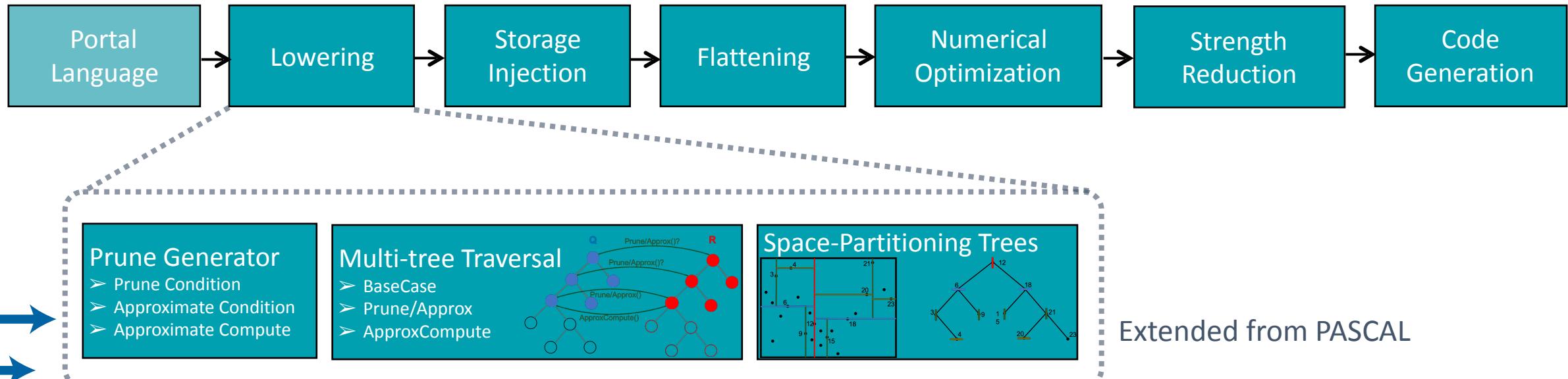
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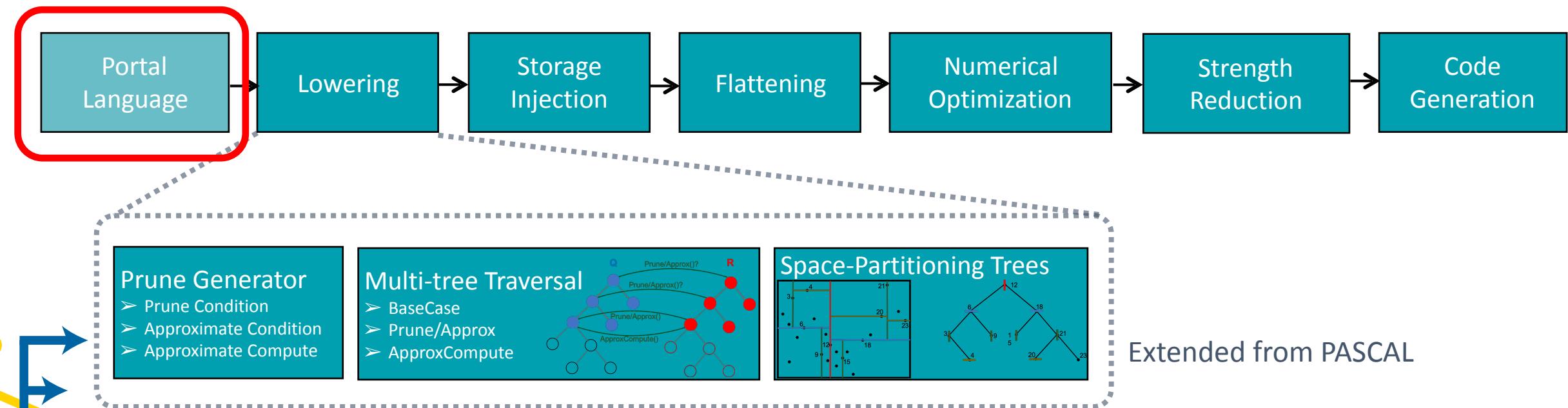
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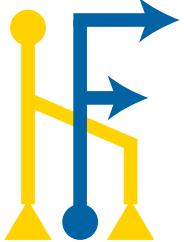
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# Portal DSL

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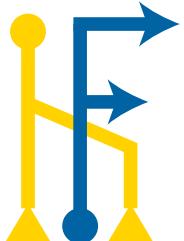
# Portal DSL

---

- Inspired by mathematical formulation

$$op_1, \dots, op_m \ K(x_1, \dots, x_m) \quad [\text{Applied to } m \text{ datasets } (D_1, \dots, D_m)]$$

- Build up an N-body problem by chaining **Layers**





# Portal DSL

---

- Inspired by mathematical formulation

$$op_1, \dots, op_m \ K(x_1, \dots, x_m) \quad [\text{Applied to } m \text{ datasets } (D_1, \dots, D_m)]$$

- Build up an N-body problem by chaining **Layers**
- Each **Layer** includes:
  - Operator
  - Dataset
  - Kernel/Modifying function



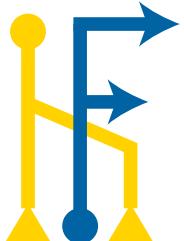


# Portal DSL

---

- Example: Nearest Neighbor

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$





# Portal DSL

---

- Example: Nearest Neighbor

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

Layer1  
Operator:  $\forall$   
Dataset:  $Q$   
Kernel: --





# Portal DSL

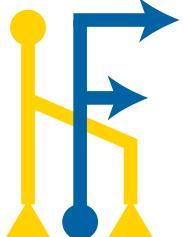
---

- Example: Nearest Neighbor

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

Layer1                          Layer2

Operator:  $\forall$                 Operator:  $\arg \min$   
Dataset:  $Q$                     Dataset:  $R$   
Kernel: --                      Kernel:  $\|x_q - x_r\|$





# Portal DSL

---

- Example: Nearest Neighbor

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```
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Storage reference("reference_file.csv");
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expr.execute();
Storage output = expr.getOutput();
```





# Portal DSL

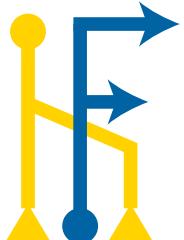
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Defining storage for each data set





# Portal DSL

---

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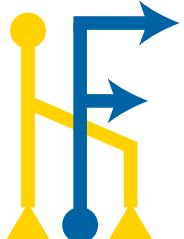
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Defining the N-body structure





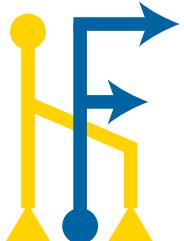
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# Portal DSL

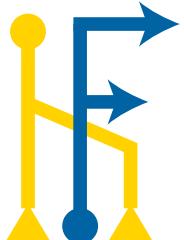
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Adding Layer 1 for NN





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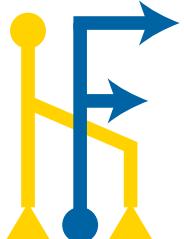
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Adding Layer 2 for NN





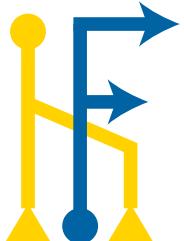
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# Portal DSL

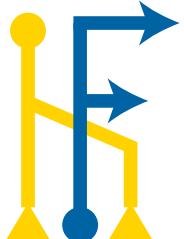
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Executing the N-body computation





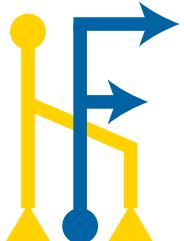
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Returning the output





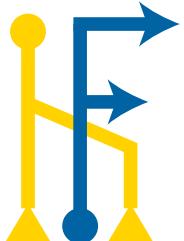
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# Portal DSL

---





# Portal DSL

- Portal operators
  - Single variable reduction
  - Multi variable reduction
  - All

Category	Math Operator	Portal Operator
Single	$\sum$	SUM
Single	$\prod$	PROD
Single	$\text{argmin}$	ARGMIN
Single	$\text{argmax}$	ARGMAX
Single	$\min$	MIN
Single	$\max$	MAX
Multi	$\cup$	UNION
Multi	$\cup \text{arg}$	UNIONARG
Multi	$\text{argmin}^k$	KARGMIN
Multi	$\text{argmax}^k$	KARGMAX
Multi	$\min^k$	KMIN
Multi	$\max^k$	KMAX
All	$\forall$	FORALL





# Portal DSL

- Portal operators
  - Single variable reduction
  - Multi variable reduction
  - All
- Storage
  - From file
  - From C++ data-structure

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# Portal DSL

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All	$\nabla$	FORALL

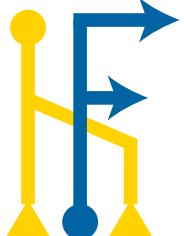




# Portal DSL

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All	$\nabla$	FORALL





# Portal DSL

---

- Portal operators
  - Single variable reduction
  - Multi variable reduction
  - All
- Storage
  - From file

```
// Construct Storage CVS file
Storage file("query_file.csv");
```

- From C++ data-structure

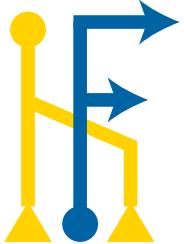
```
// Construct Storage from C++ data-structure
std::vector<std::vector<float>> input;
// Fill in the input data structure with values
Storage query(input);
```





# Portal DSL

---





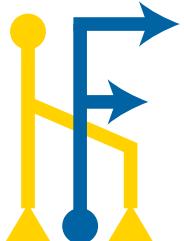
# Portal DSL

---

- Kernel function
  - Pre-defined

```
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MANHATTAN) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::CHEBYSHEV) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MAHALANOBIS) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::SQREUCDIST) ;
```

- User-defined (nearest neighbor example)





# Portal DSL

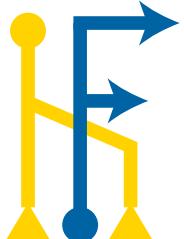
---

- Kernel function
  - Pre-defined

Name of the predefined kernel function

```
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MANHATTAN));  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::CHEBYSHEV));  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MAHALANOBIS));  
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- User-defined (nearest neighbor example)





# Portal DSL

---

- Kernel function
  - Pre-defined

```
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MANHATTAN) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::CHEBYSHEV) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MAHALANOBIS) ;  
expr.addLayer(PortalOp::FORALL, ref, PortalFunc::SQREUCDIST) ;
```

- User-defined (nearest neighbor example)





# Portal DSL

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```

- User-defined (nearest neighbor example)

```
Storage query("query_file.csv");
Storage reference("reference_file.csv");
Var q,r;
Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL, q, query);
expr.addLayer(PortalOp::ARGMIN, r, reference, EuclidDist);
expr.execute();
Storage output = expr.getOutput();
```





# Portal DSL

---

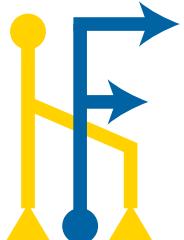
- Kernel function
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```

Var q, r; Keyword for defining variables





# Portal DSL

---

- Kernel function
  - Pre-defined

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# Portal DSL

---

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  - Pre-defined

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User-defined kernel function





# Portal DSL

---

- Kernel function
  - Pre-defined

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expr.addLayer(PortalOp::FORALL, ref, PortalFunc::MANHATTAN));
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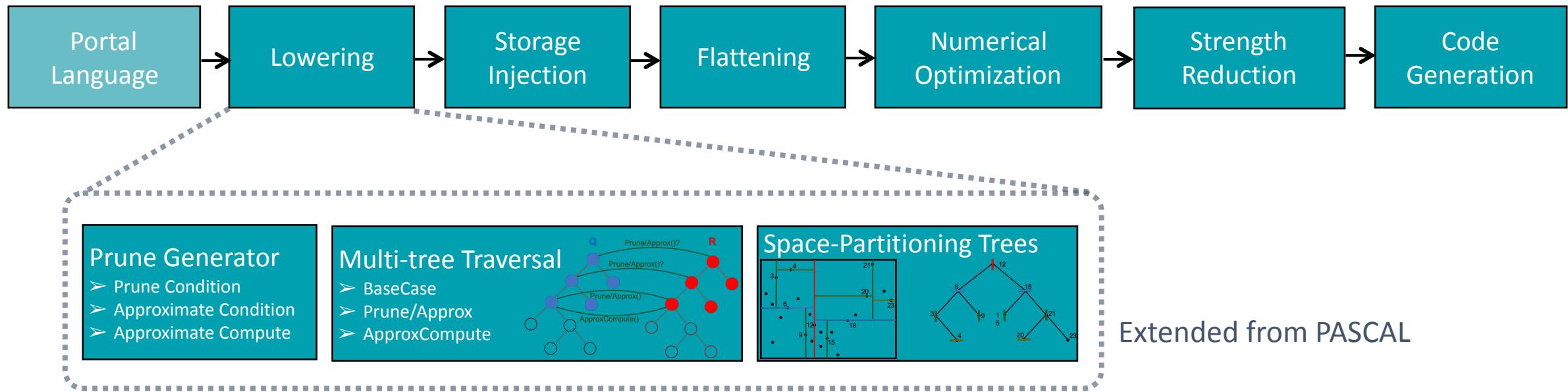
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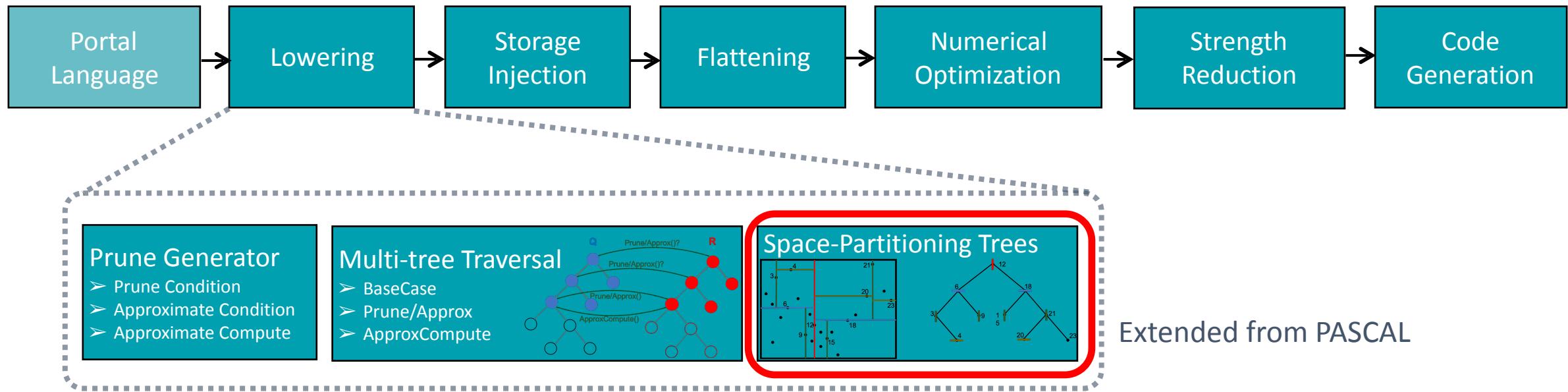


# Portal





# Portal





# Space-partitioning Tree

---

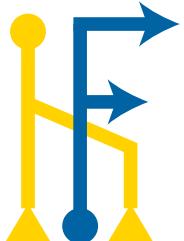




# Space-partitioning Tree

---

- K-d tree

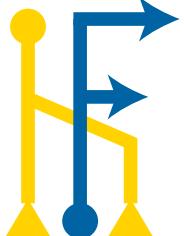
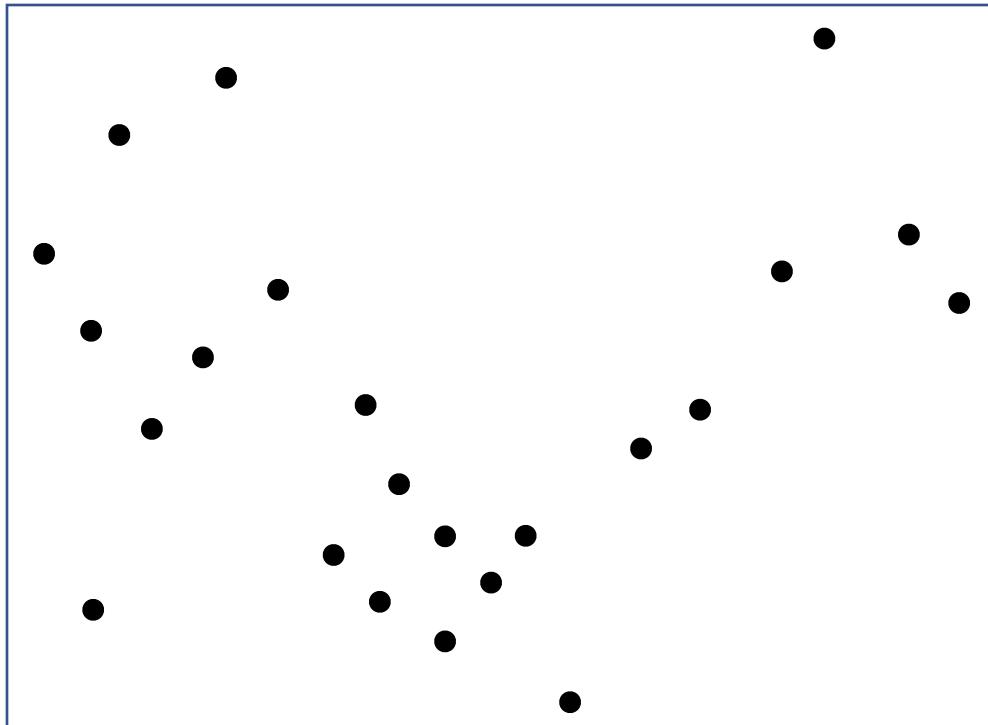




# Space-partitioning Tree

---

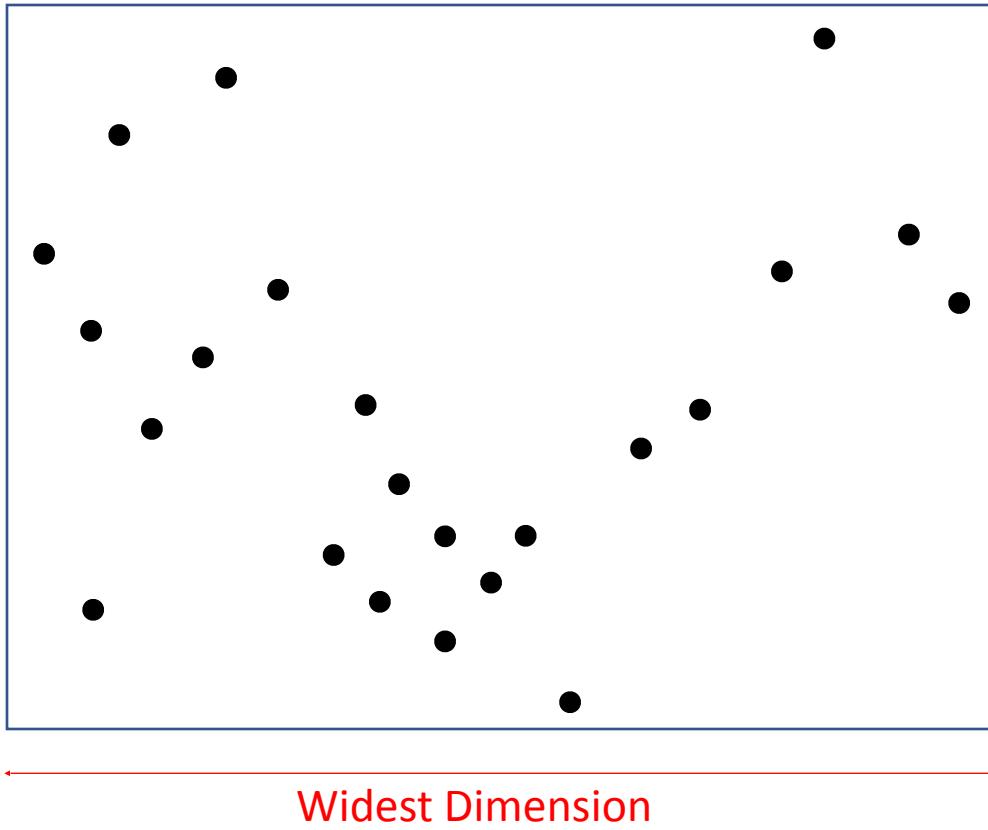
- K-d tree





# Space-partitioning Tree

- K-d tree

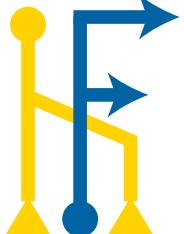
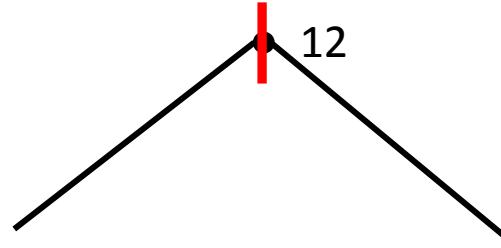
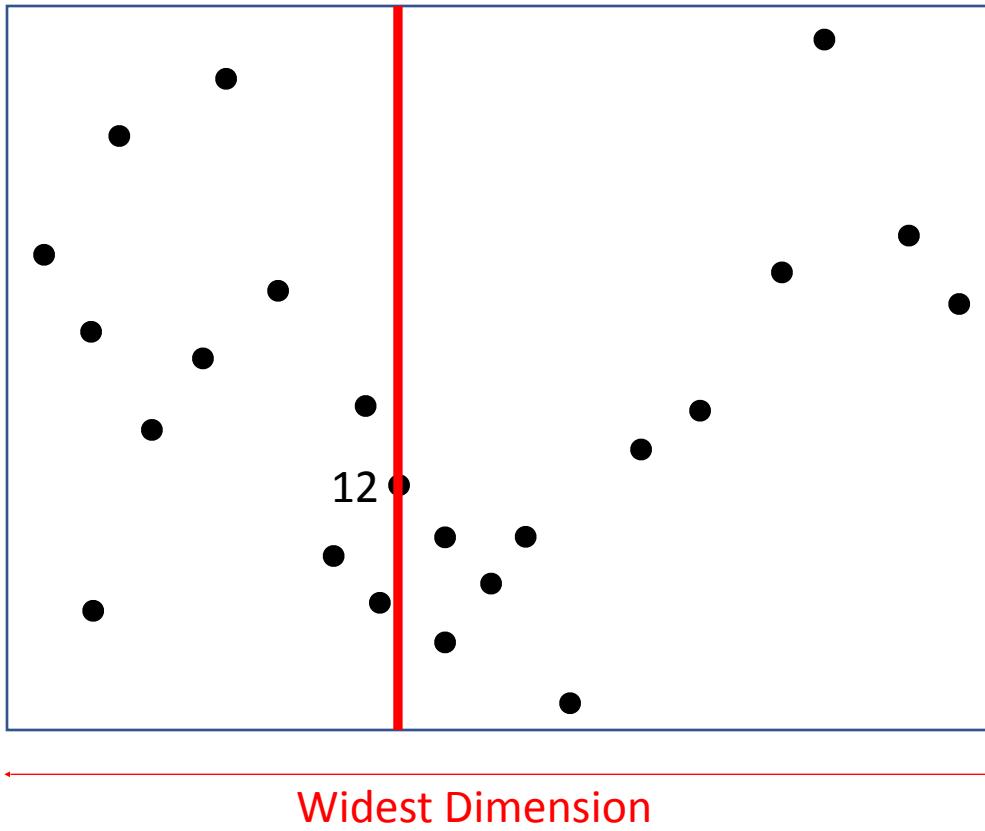




# Space-partitioning Tree

- K-d tree

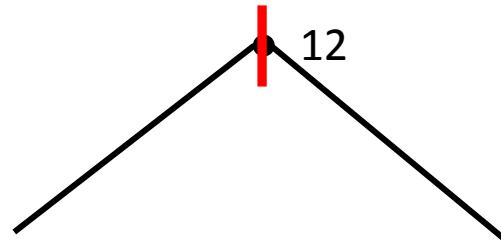
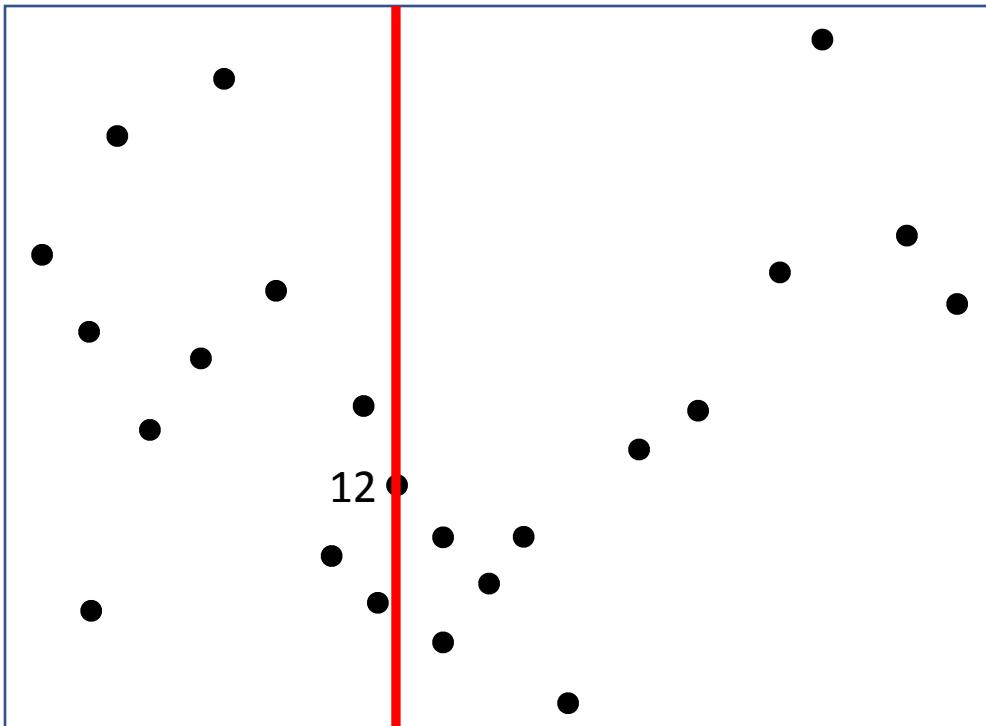
Median Partitioning





# Space-partitioning Tree

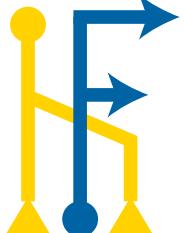
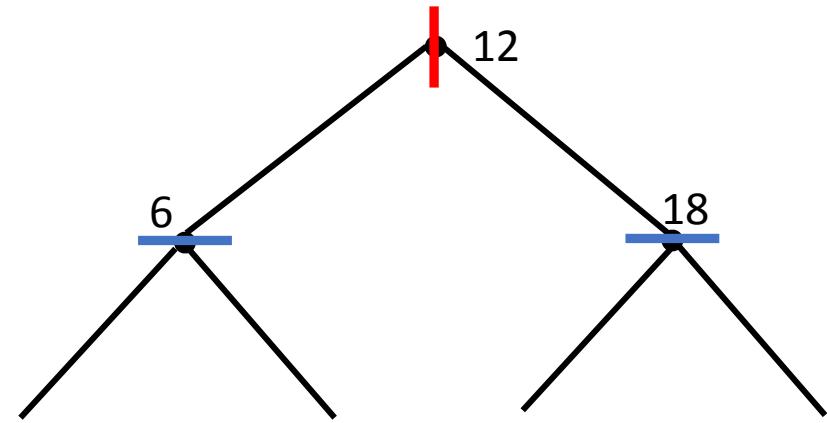
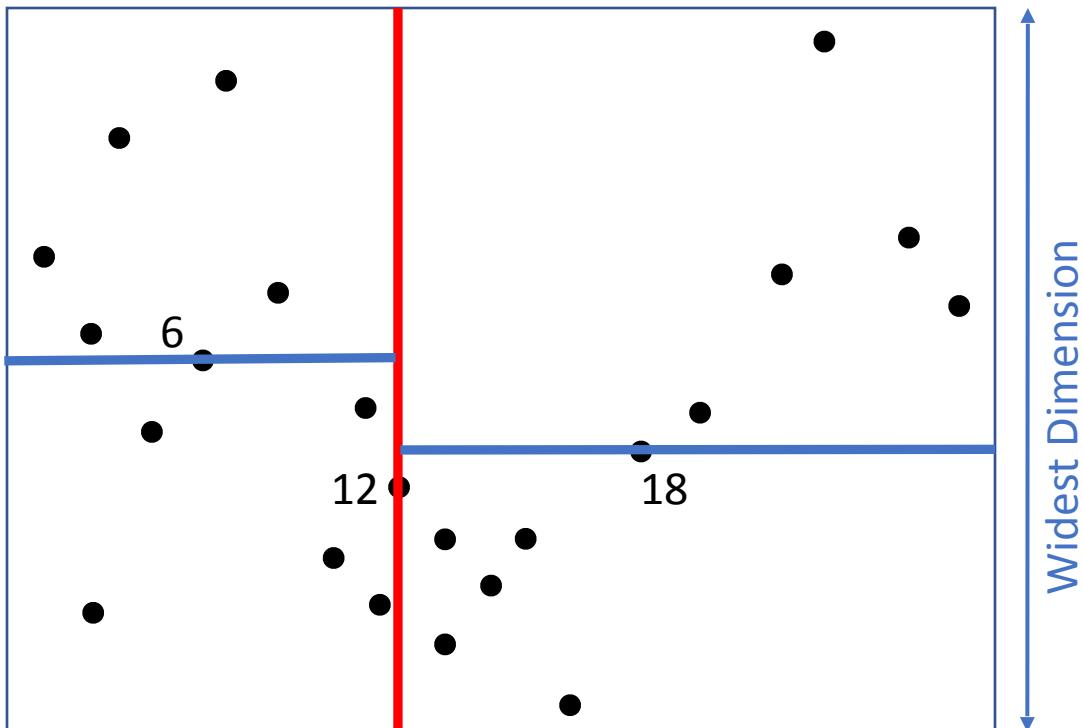
- K-d tree





# Space-partitioning Tree

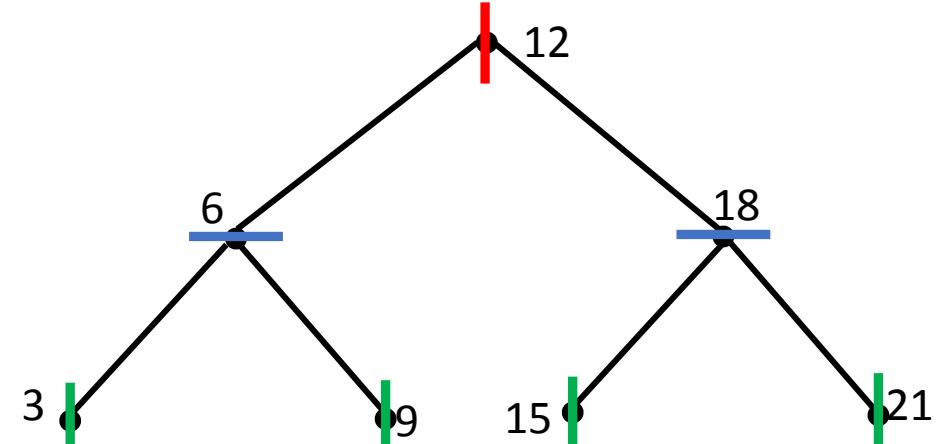
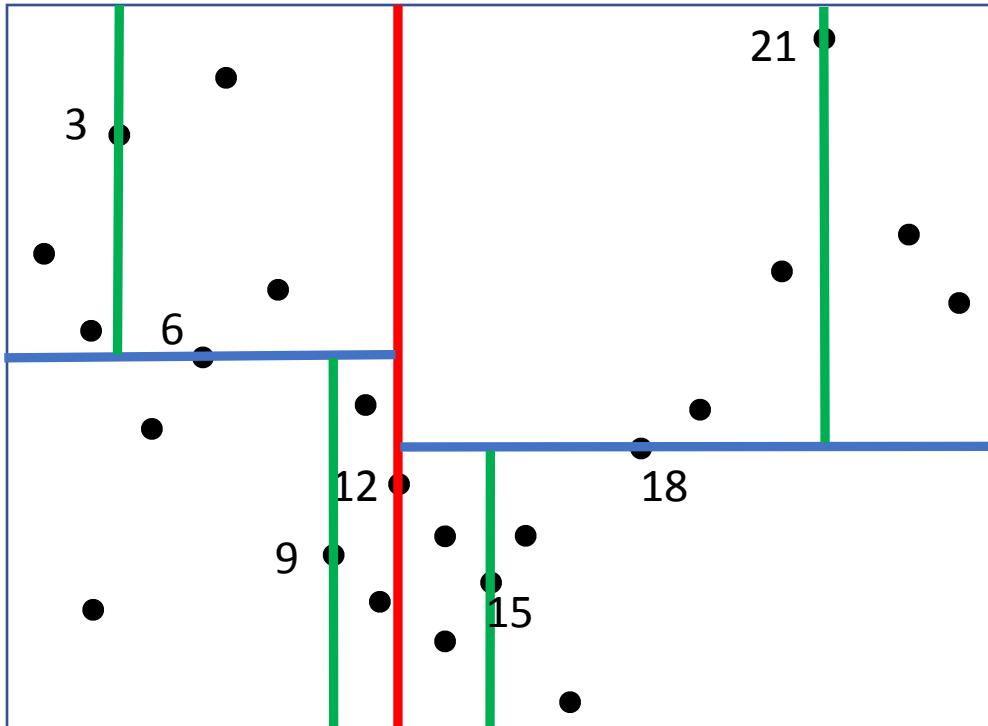
- K-d tree





# Space-partitioning Tree

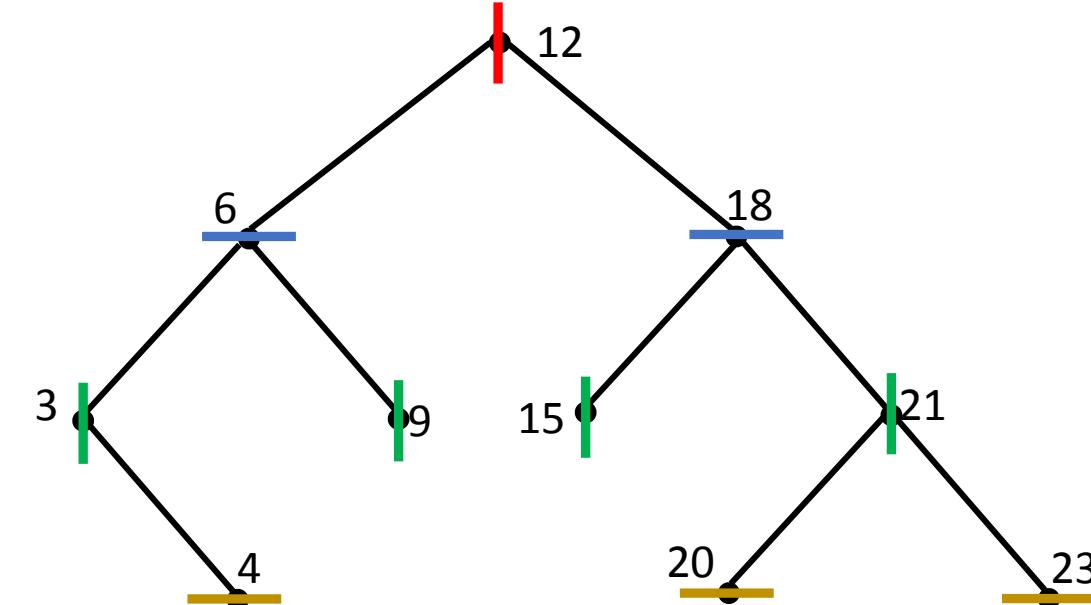
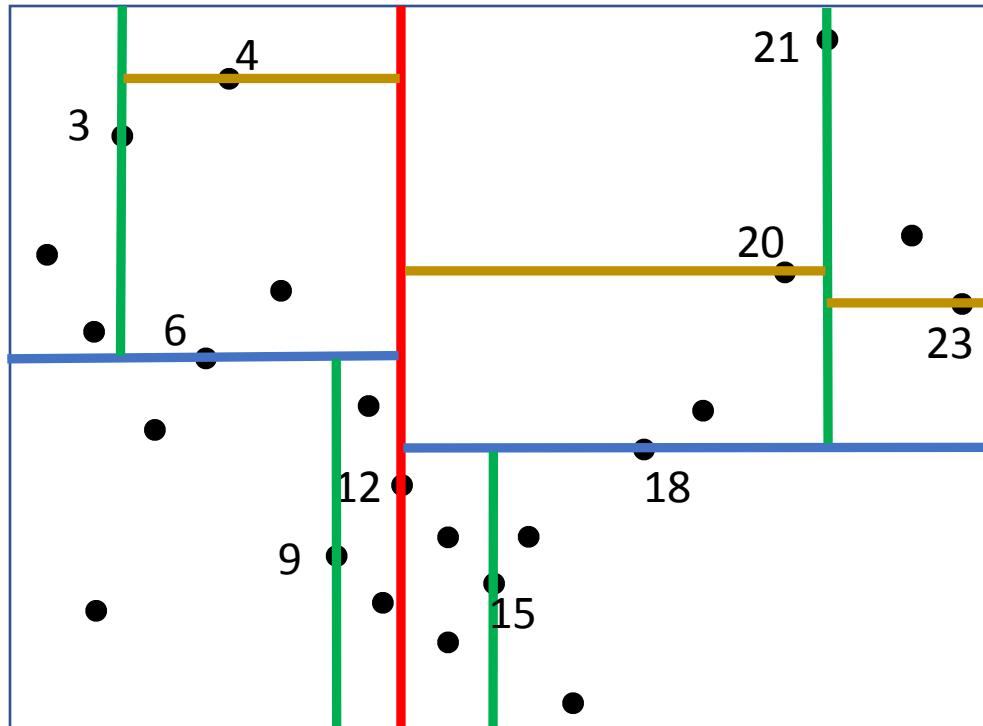
- K-d tree





# Space-partitioning Tree

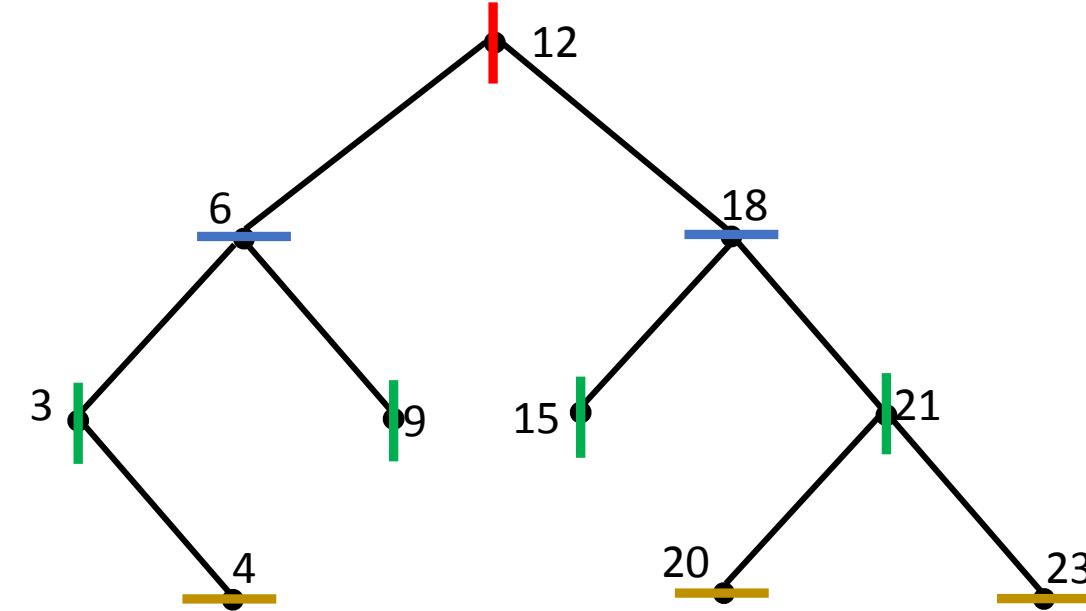
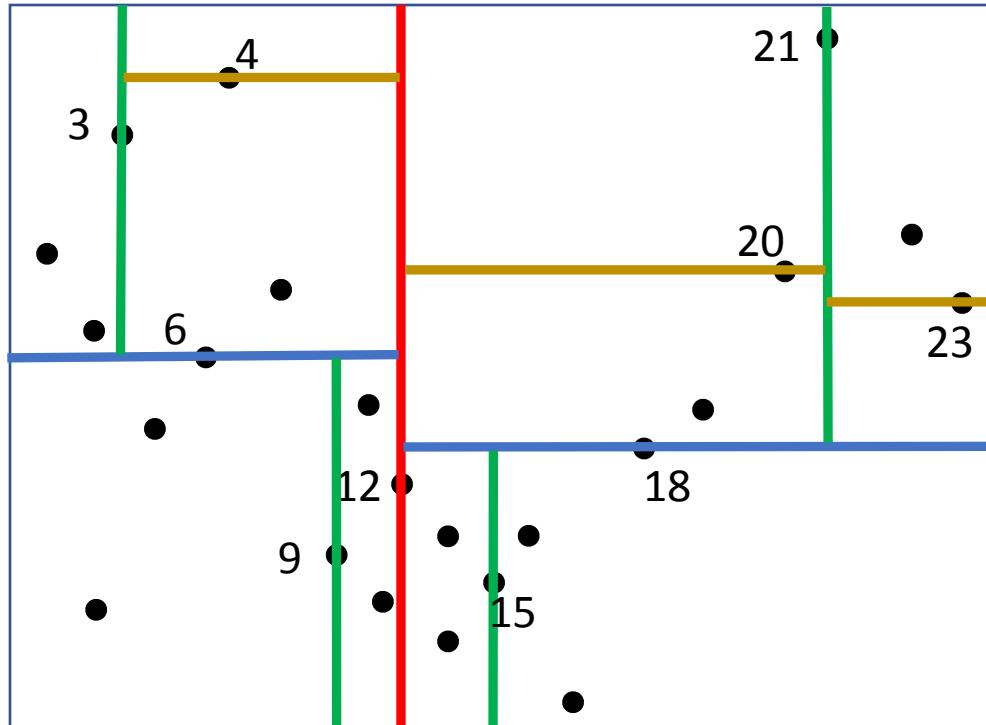
- K-d tree





# Space-partitioning Tree

- K-d tree



Recursively divide space until each box has at most  $q$  data point



# Classification of N-body Problems

---





# Classification of N-body Problems

---

- Approximation





# Classification of N-body Problems

---

- Approximation ➔ Kernel Density Estimation

$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$





# Classification of N-body Problems

---

- Approximation → Kernel Density Estimation

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Arithmetic operator      Non-comparative kernel





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Arithmetic operator

Non-comparative kernel

- Pruning





# Classification of N-body Problems

- Approximation → Kernel Density Estimation

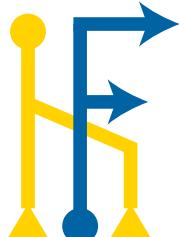
$$\forall q \in Q, \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$

Arithmetic operator

Non-comparative kernel

- Pruning → Nearest Neighbor

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$





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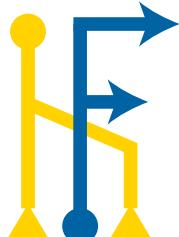
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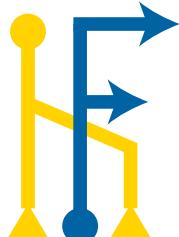
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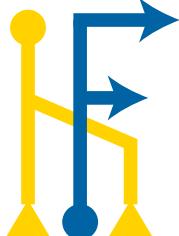
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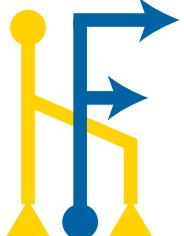
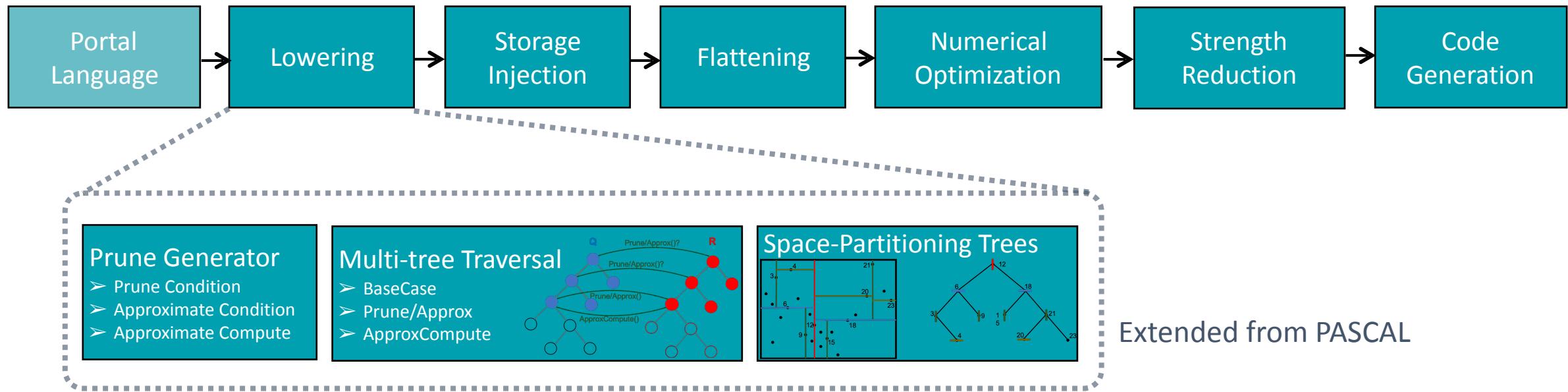
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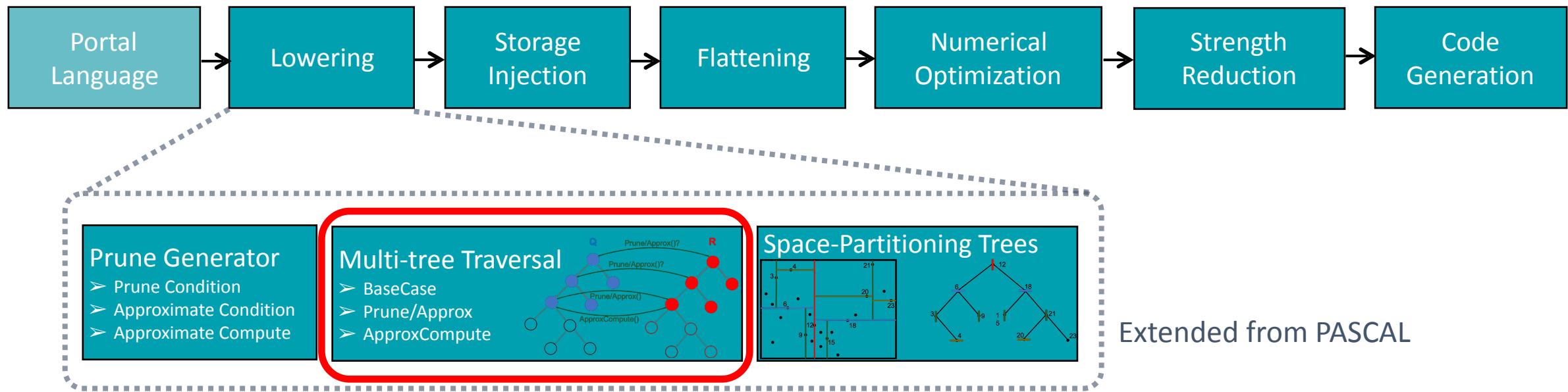


# Portal





# Portal





# Multi-tree Traversal

---

- Functionalities
  - BaseCase
  - Prune/Approx
  - ApproxCompute

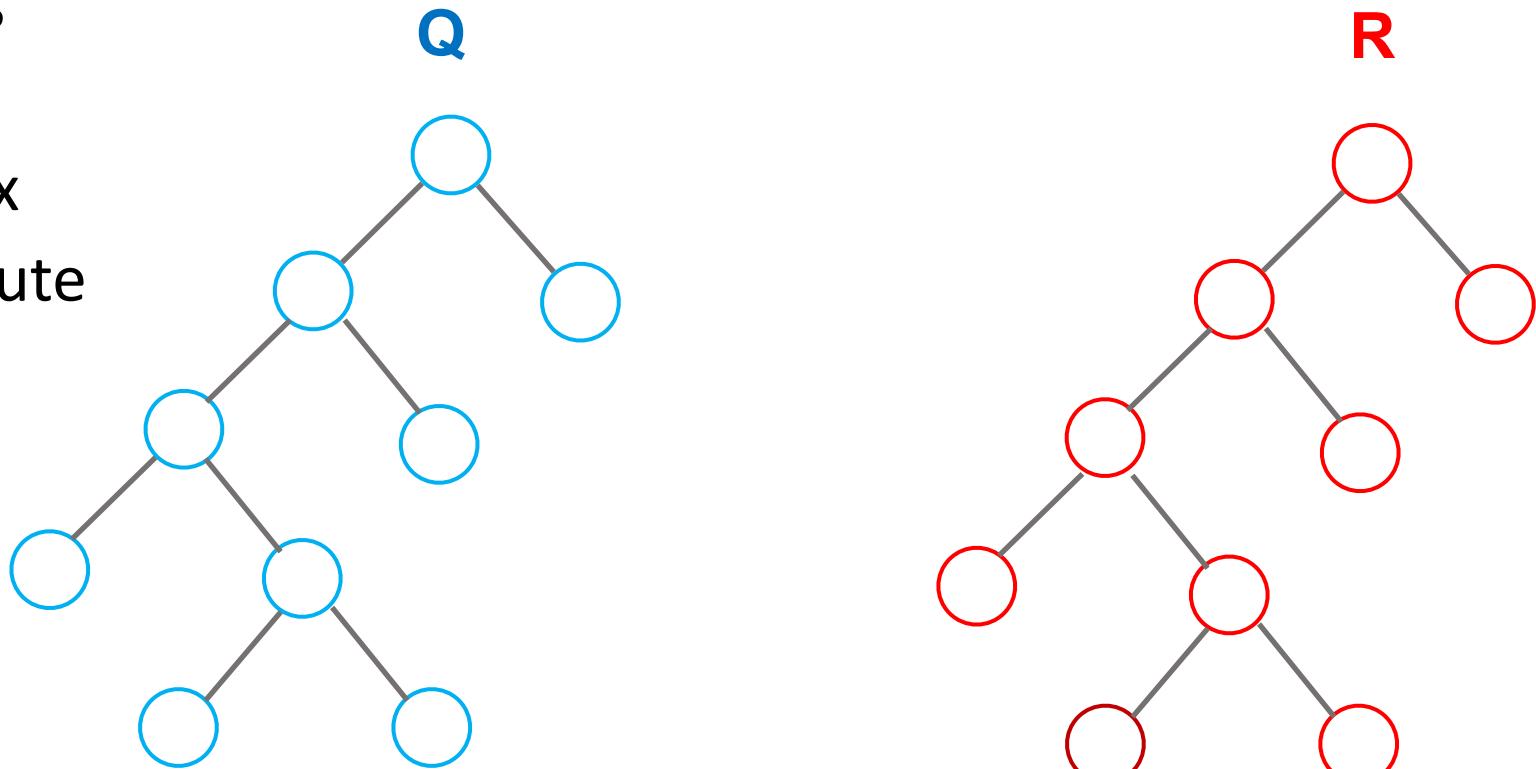




# Multi-tree Traversal

---

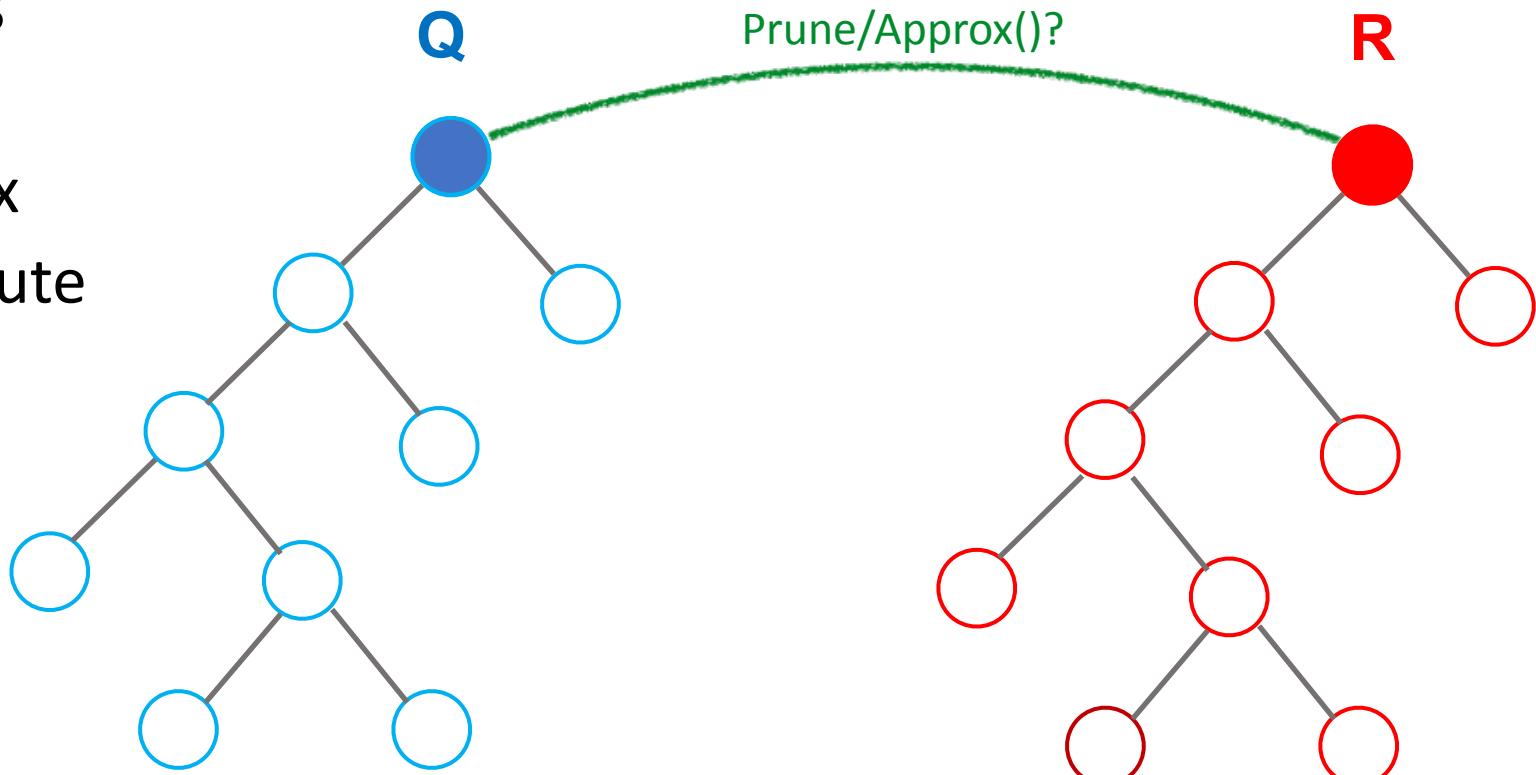
- Functionalities
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# Multi-tree Traversal

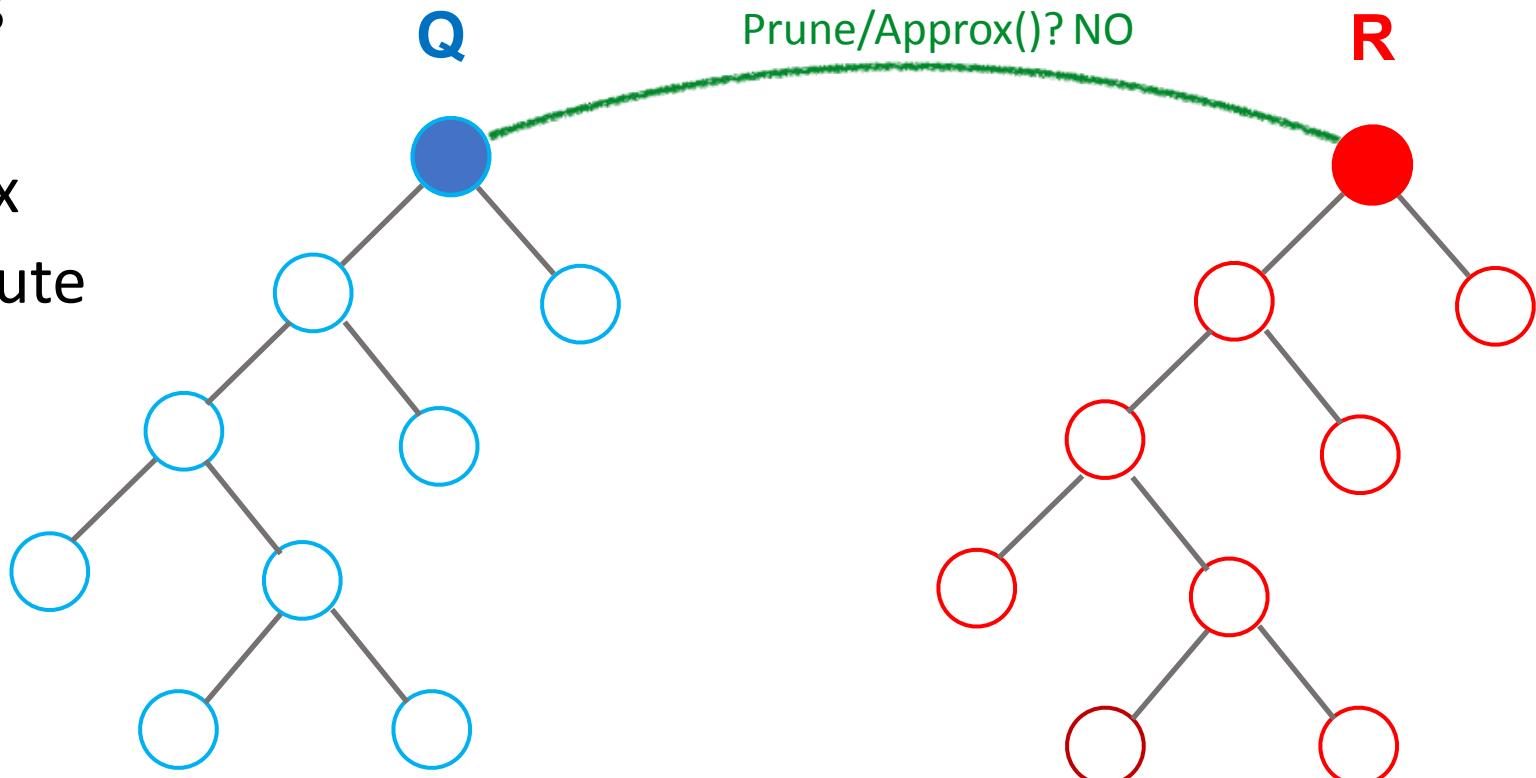
- Functionalities
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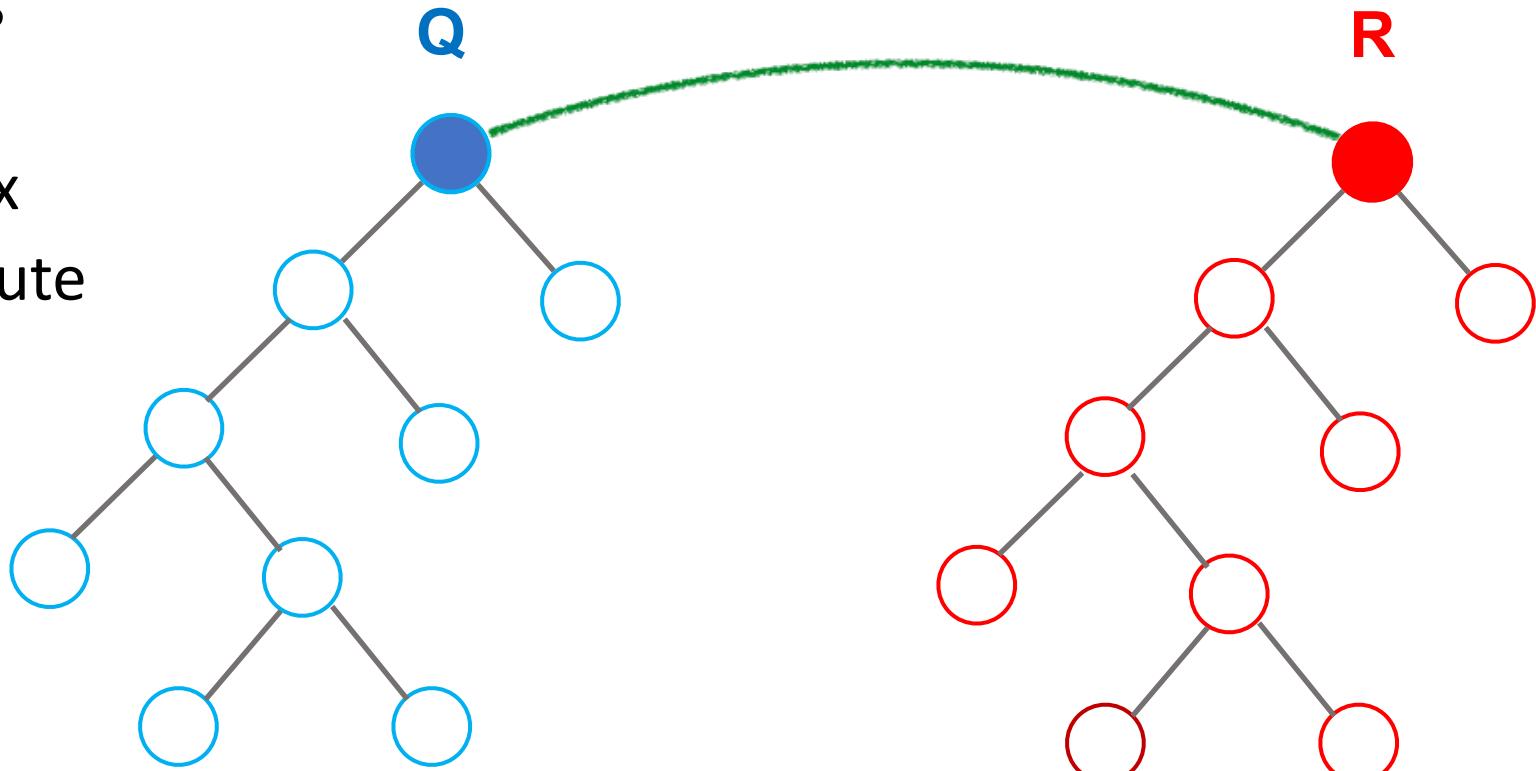
# Multi-tree Traversal

- Functionalities
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# Multi-tree Traversal

- Functionalities
  - BaseCase
  - Prune/Approx
  - ApproxCompute

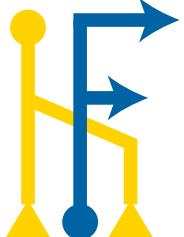
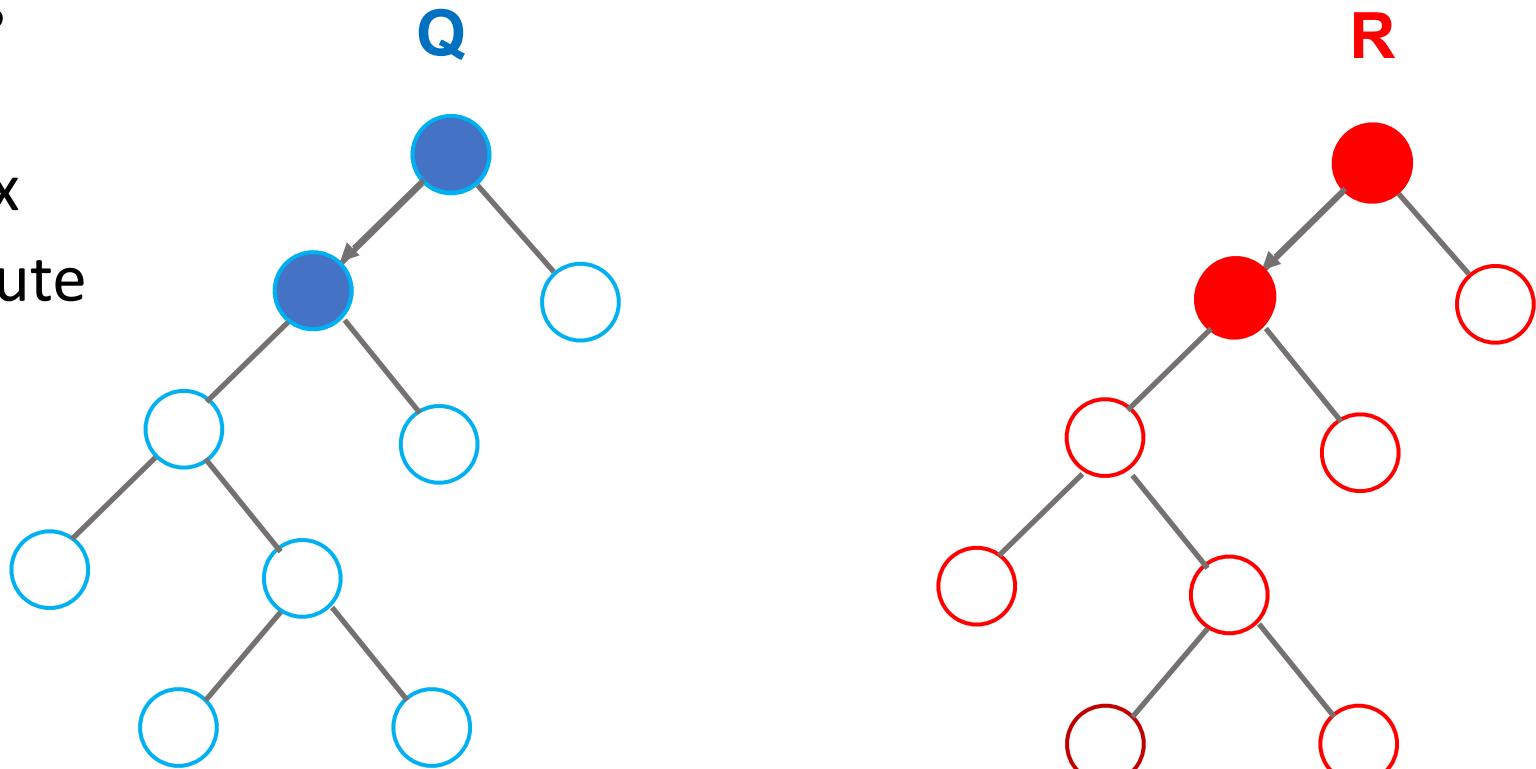




# Multi-tree Traversal

---

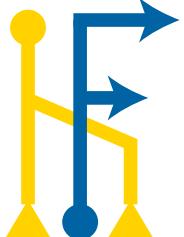
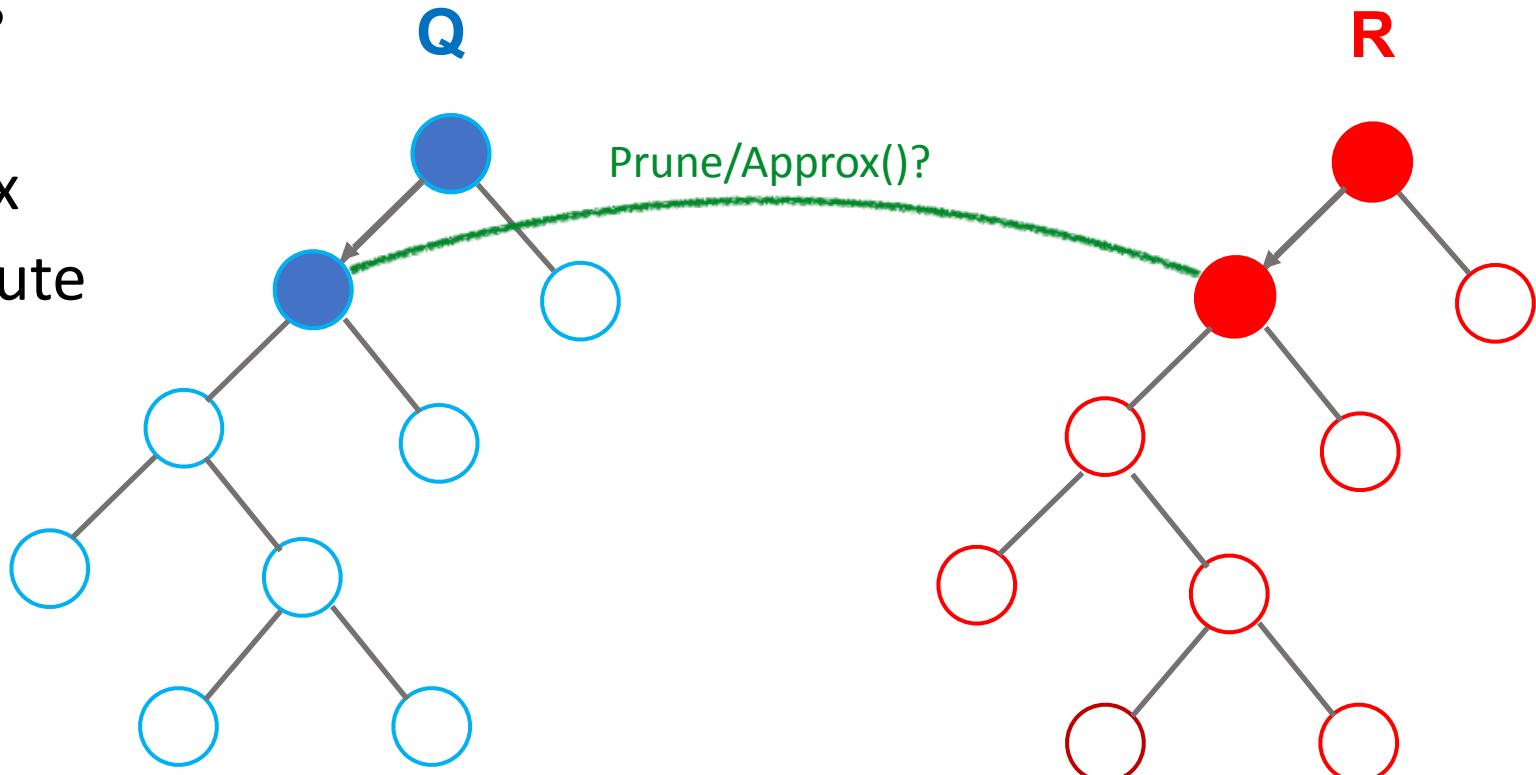
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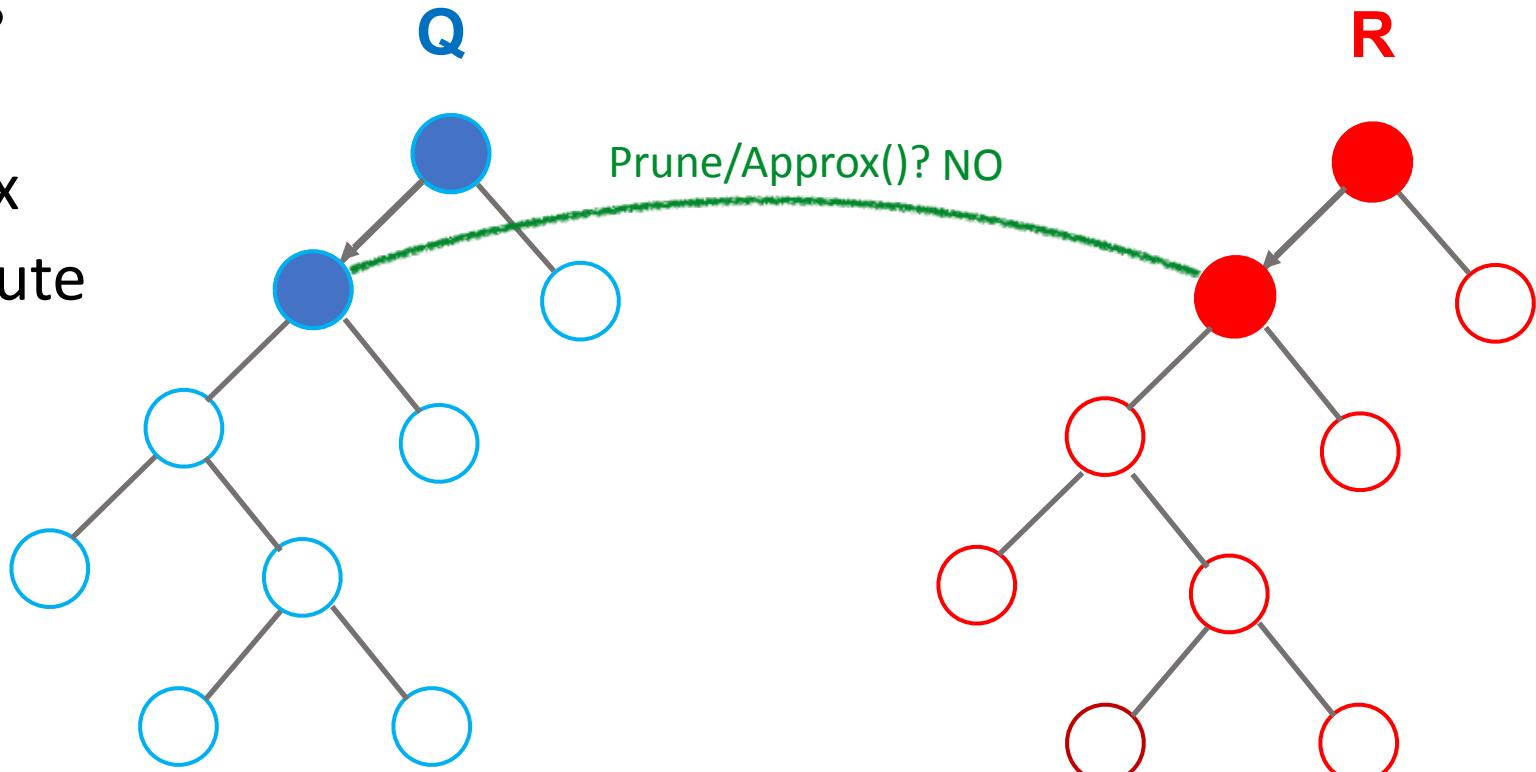
# Multi-tree Traversal

- Functionalities
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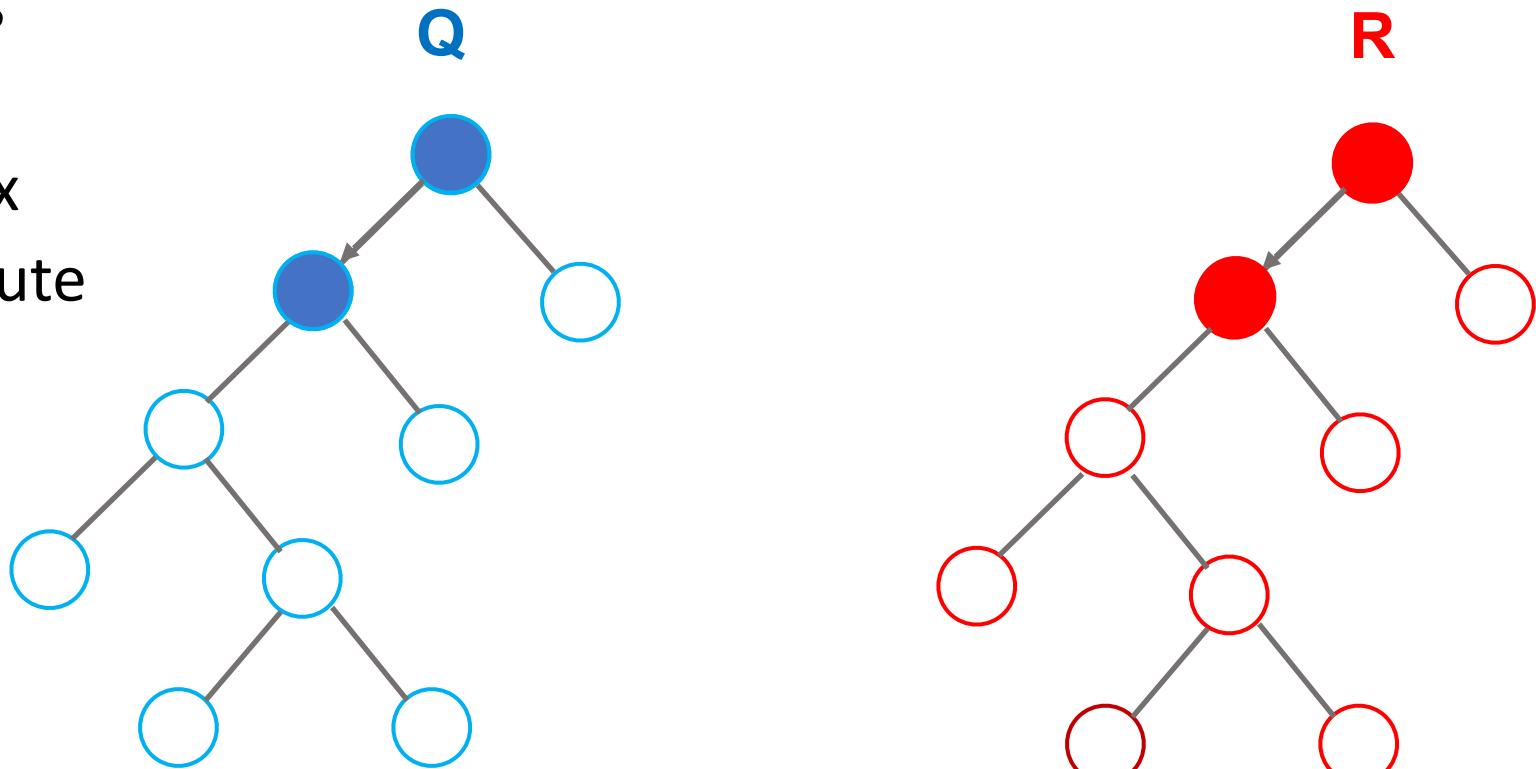




# Multi-tree Traversal

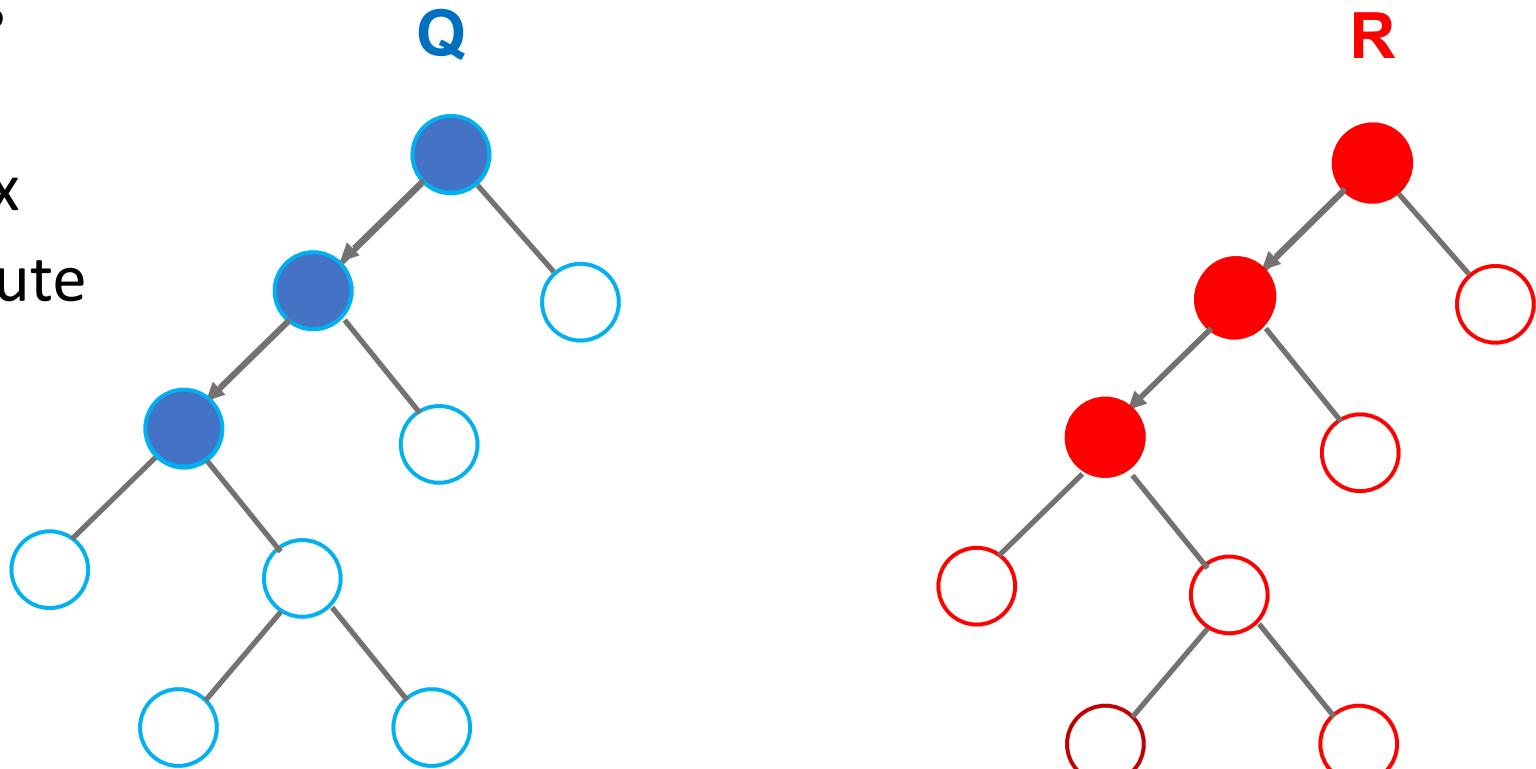
---

- Functionalities
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# Multi-tree Traversal

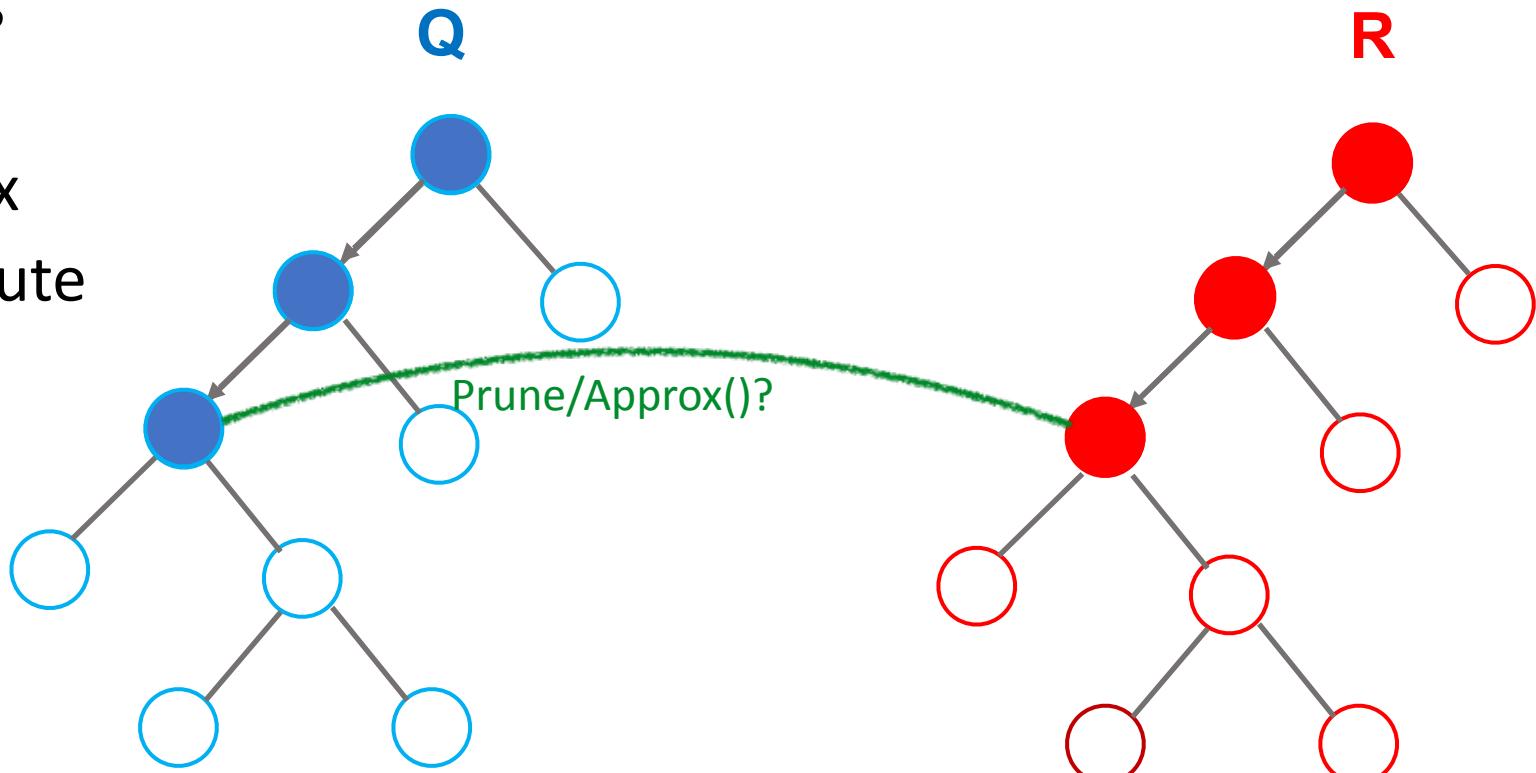
- Functionalities
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  - Prune/Approx
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# Multi-tree Traversal

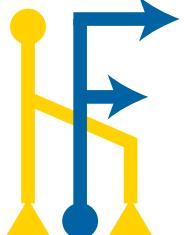
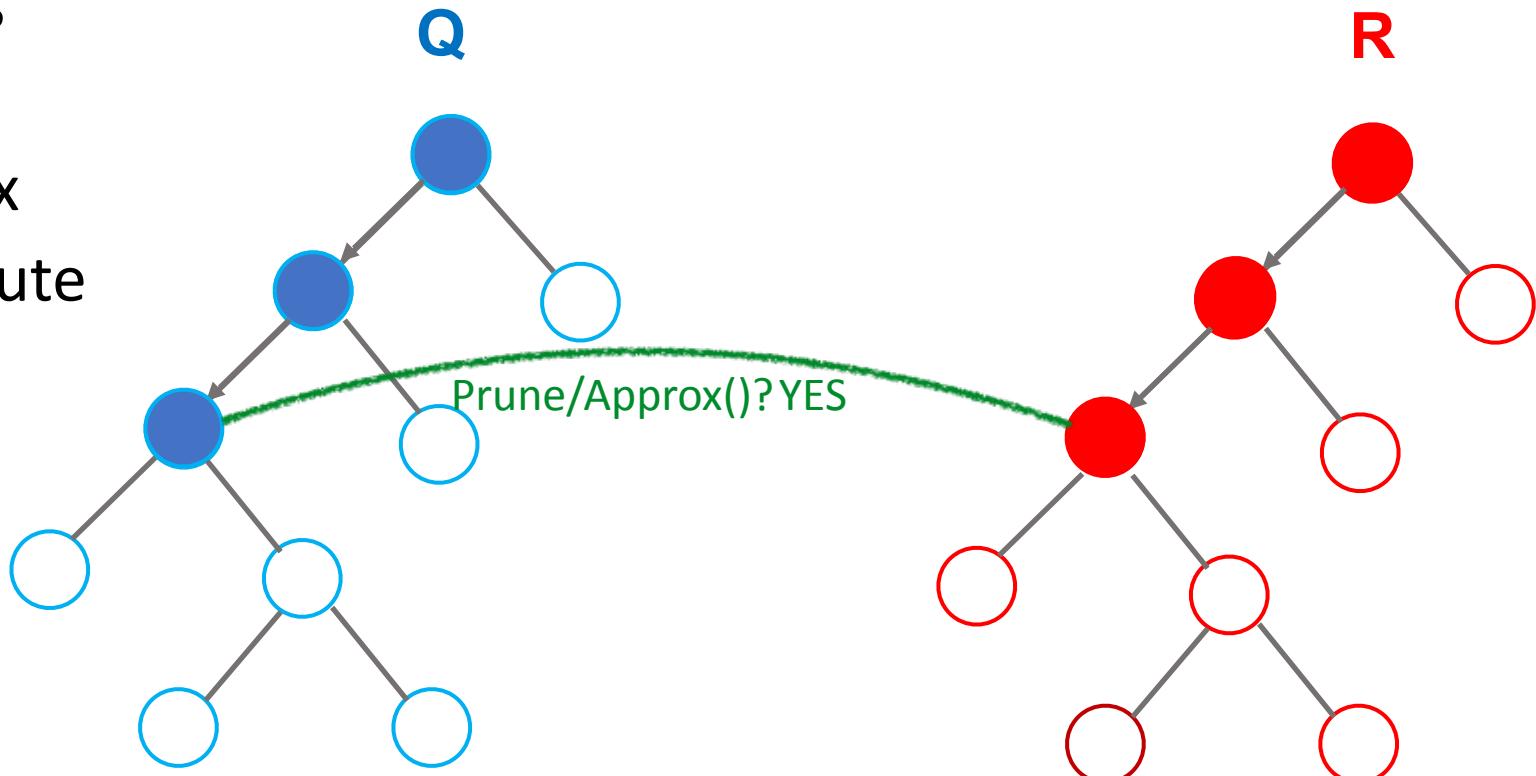
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# Multi-tree Traversal

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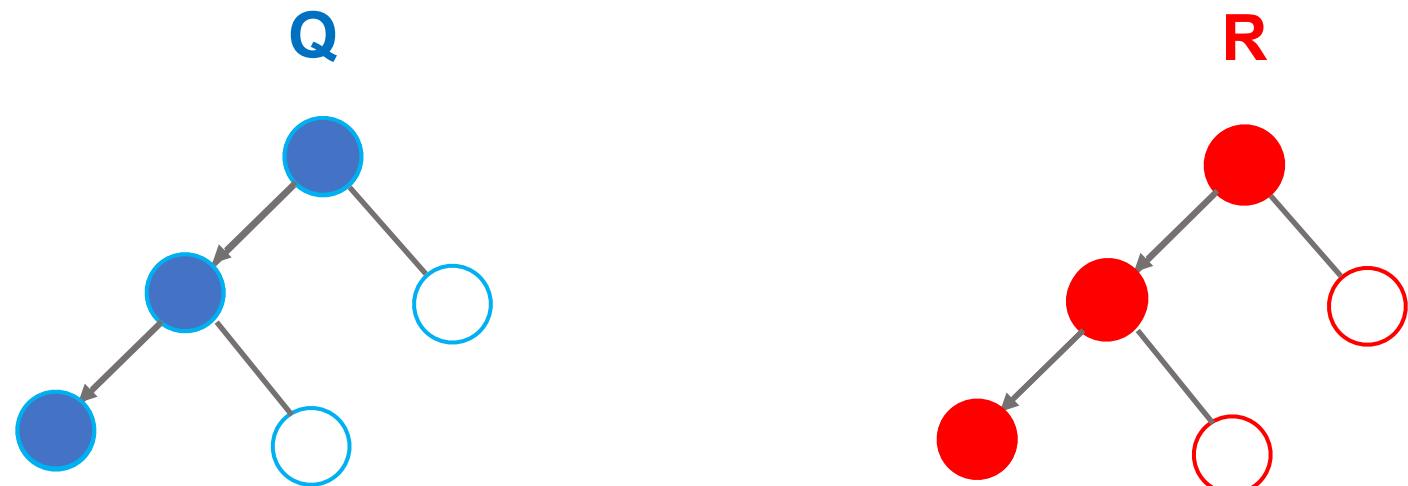




# Multi-tree Traversal

---

- Functionalities
  - BaseCase
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For pruning problems: if Prune/Approx() is true, discard the entire subtree

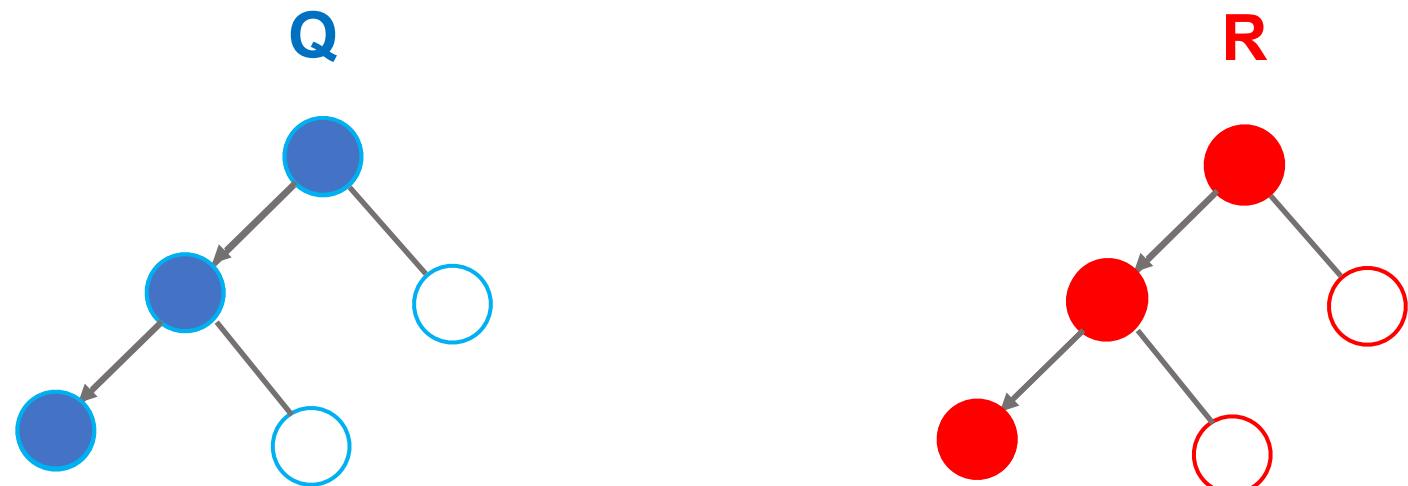




# Multi-tree Traversal

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  - BaseCase
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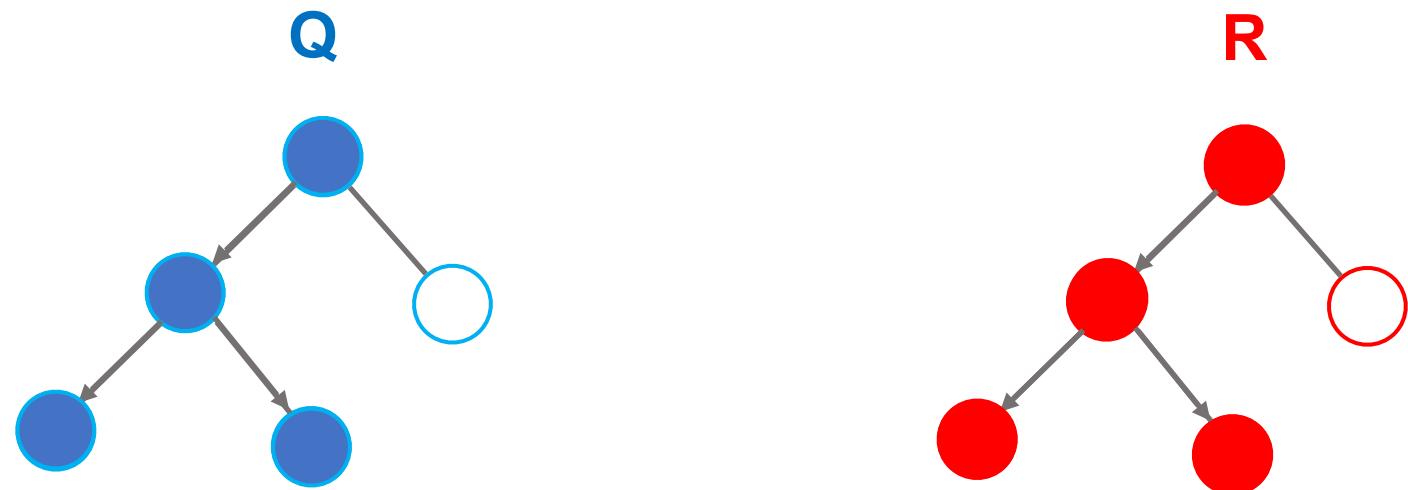




# Multi-tree Traversal

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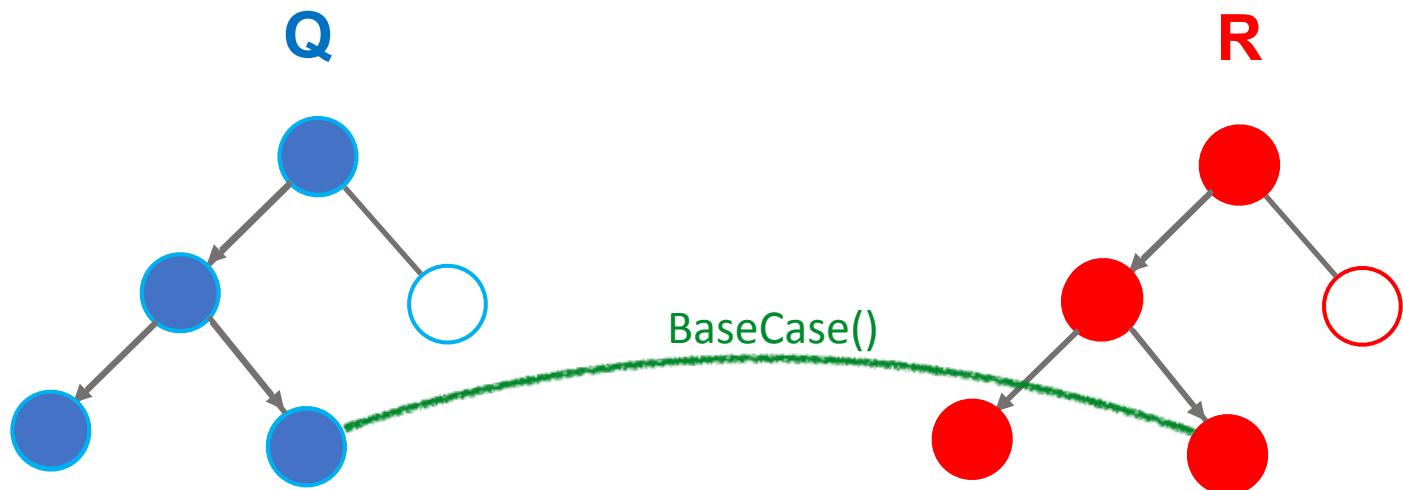
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# Multi-tree Traversal

- Functionalities
  - BaseCase
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Direct computation  $O(q^2)$

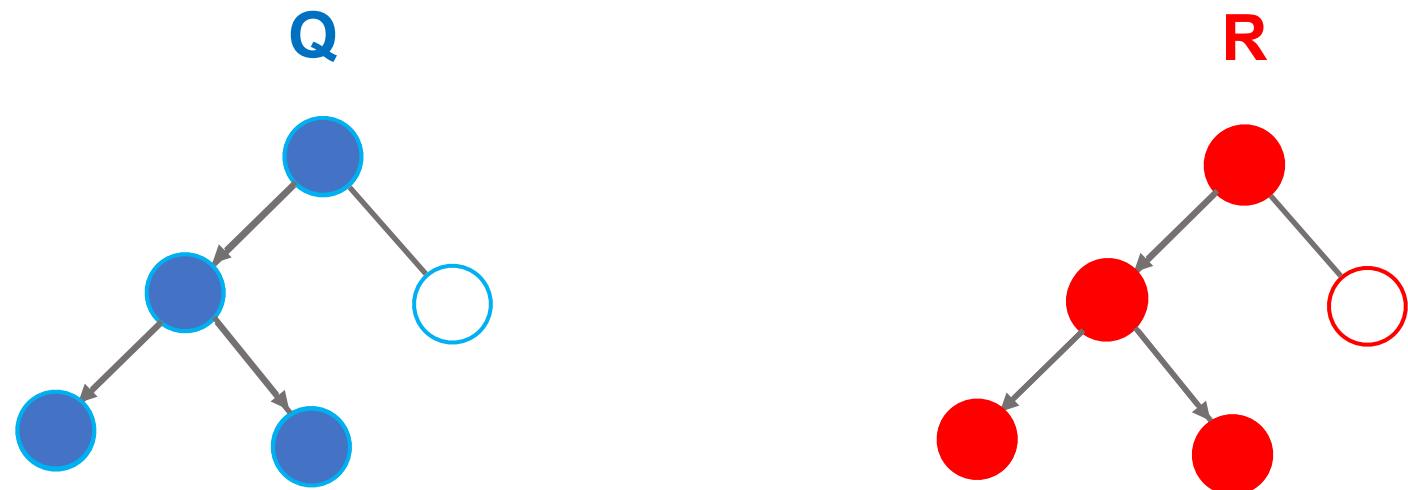




# Multi-tree Traversal

---

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  - BaseCase
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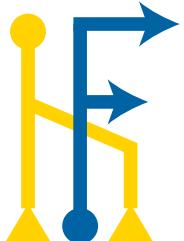
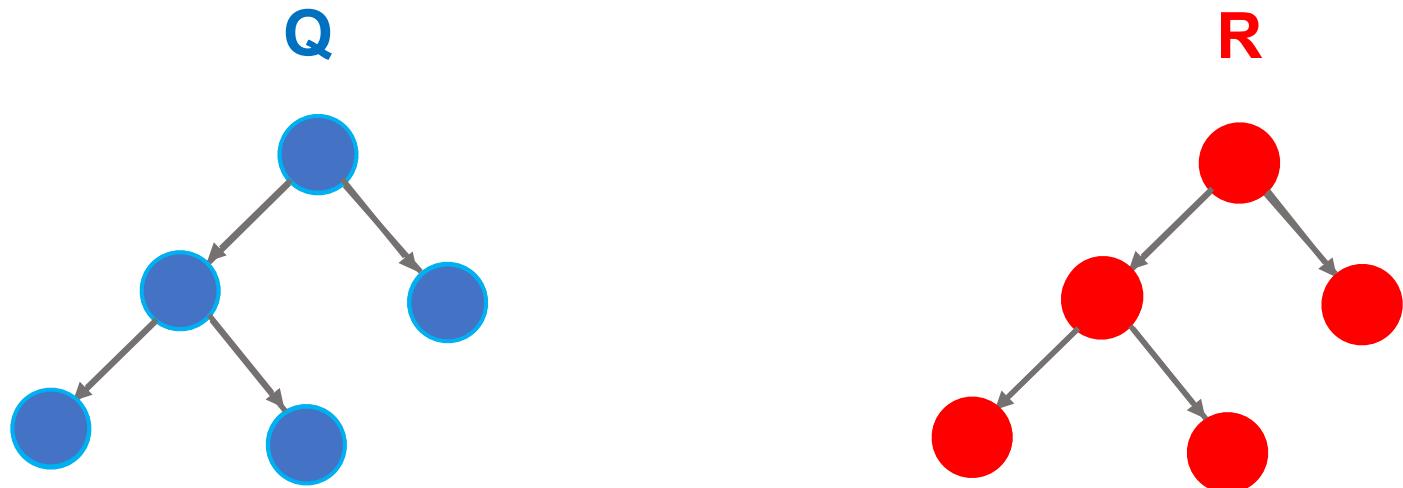




# Multi-tree Traversal

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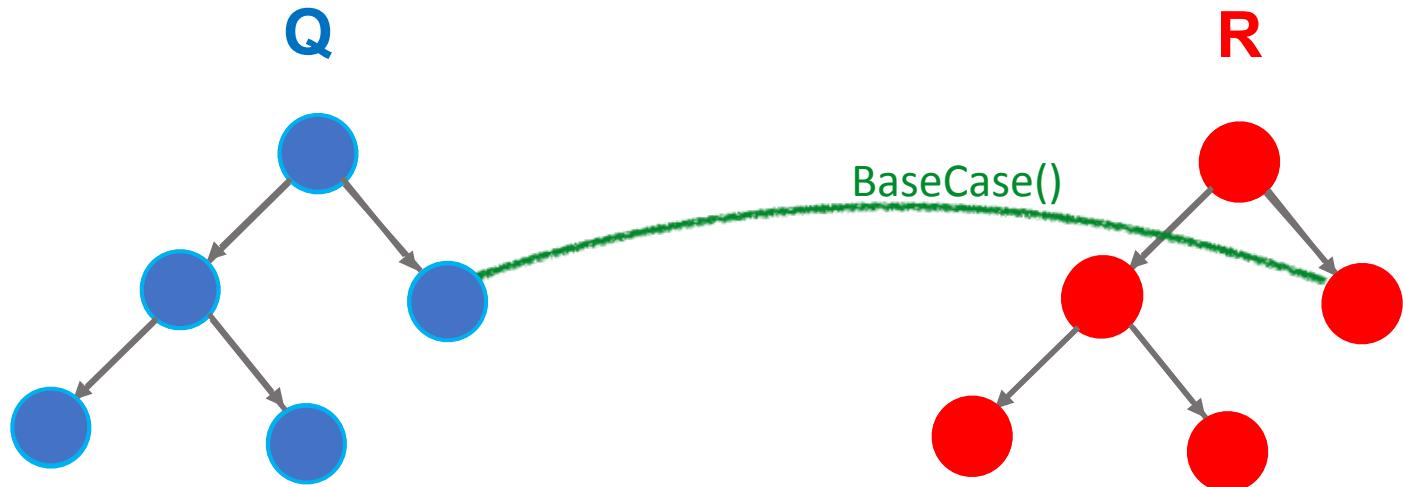
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# Multi-tree Traversal

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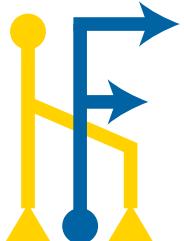
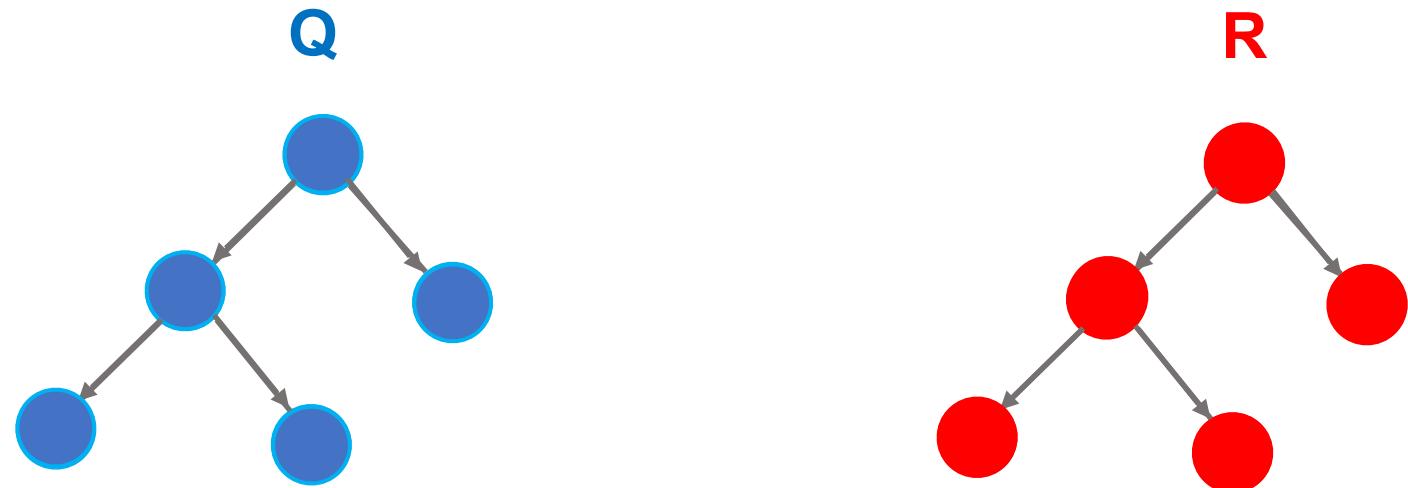




# Multi-tree Traversal

---

- Functionalities
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  - ApproxCompute

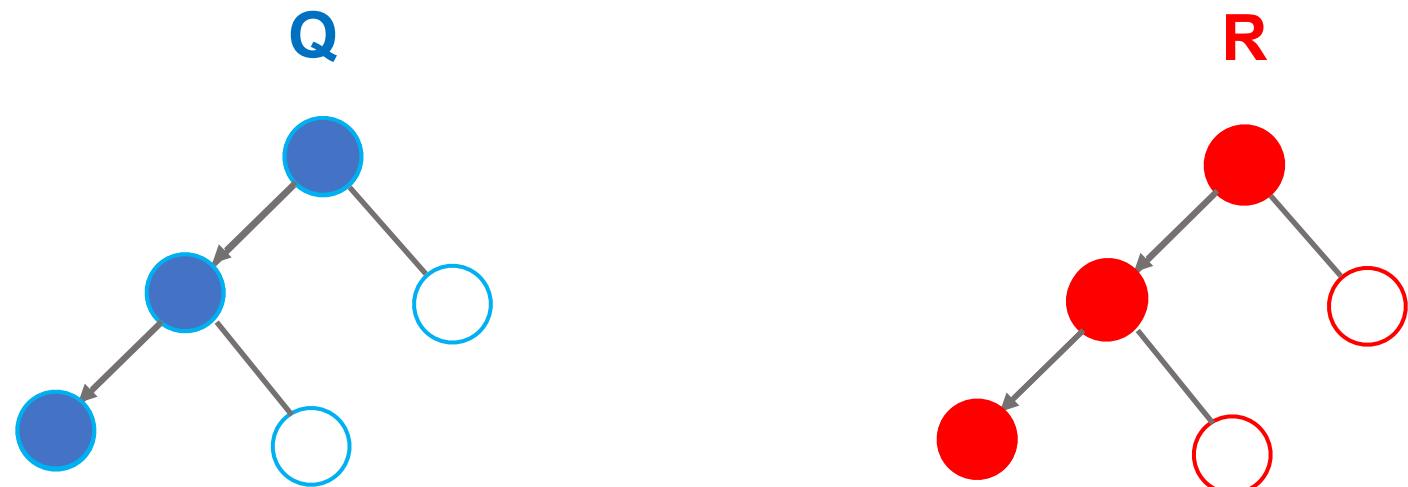




# Multi-tree Traversal

---

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  - ApproxCompute

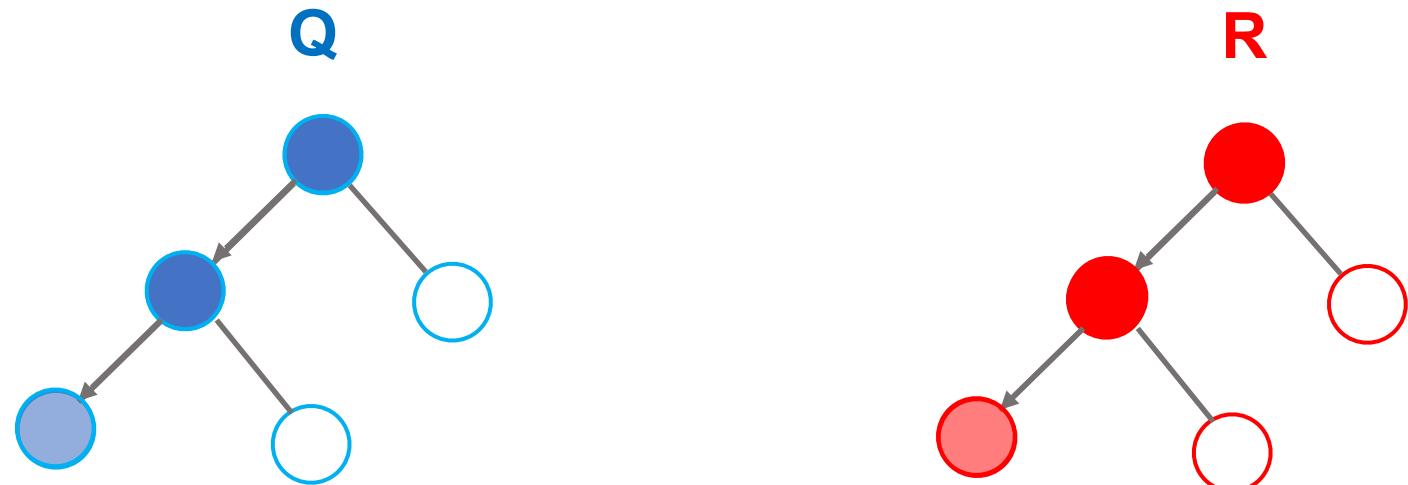




# Multi-tree Traversal

---

- Functionalities
  - BaseCase
  - Prune/Approx
  - ApproxCompute



For Approximation problems: if Prune/Approx() is true, replace the subtree with the centroid

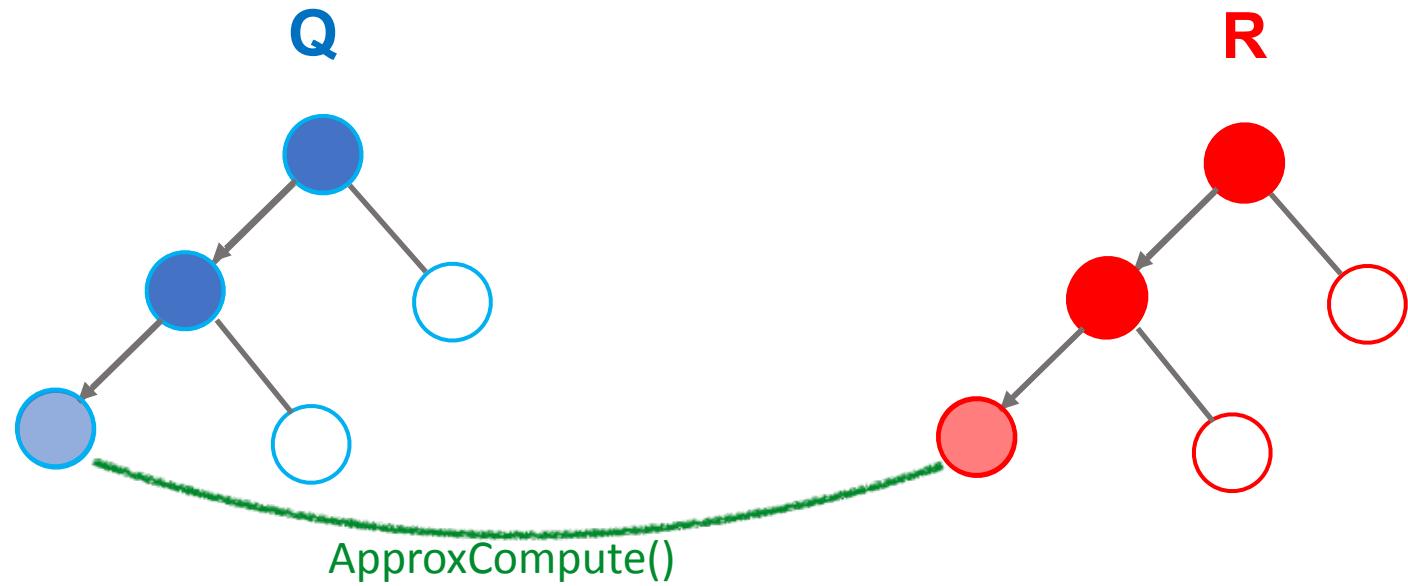




# Multi-tree Traversal

---

- Functionalities
  - BaseCase
  - Prune/Approx
  - ApproxCompute

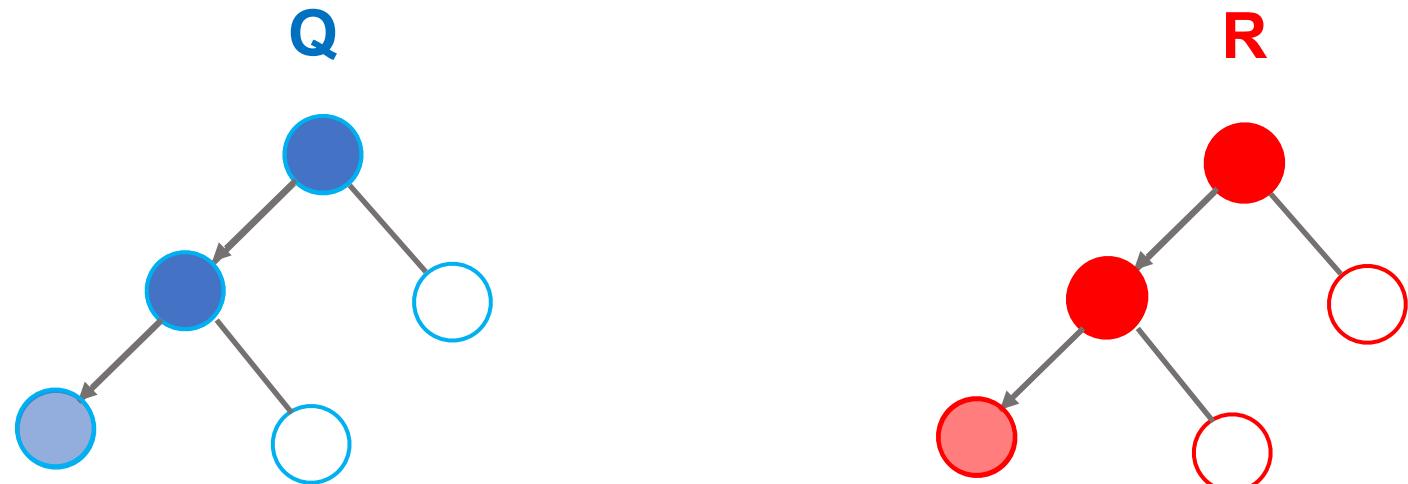




# Multi-tree Traversal

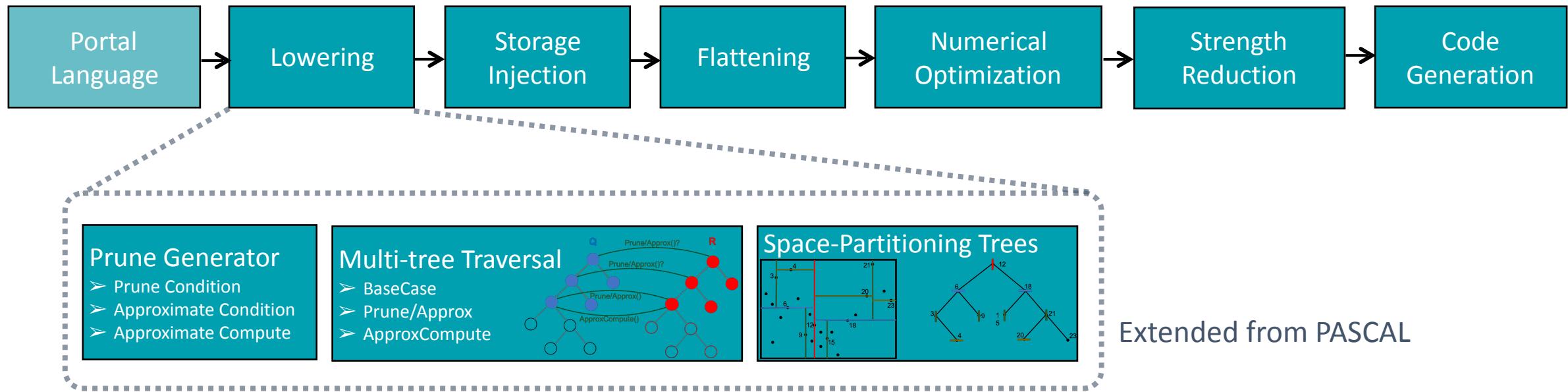
---

- Functionalities
  - BaseCase
  - Prune/Approx
  - ApproxCompute



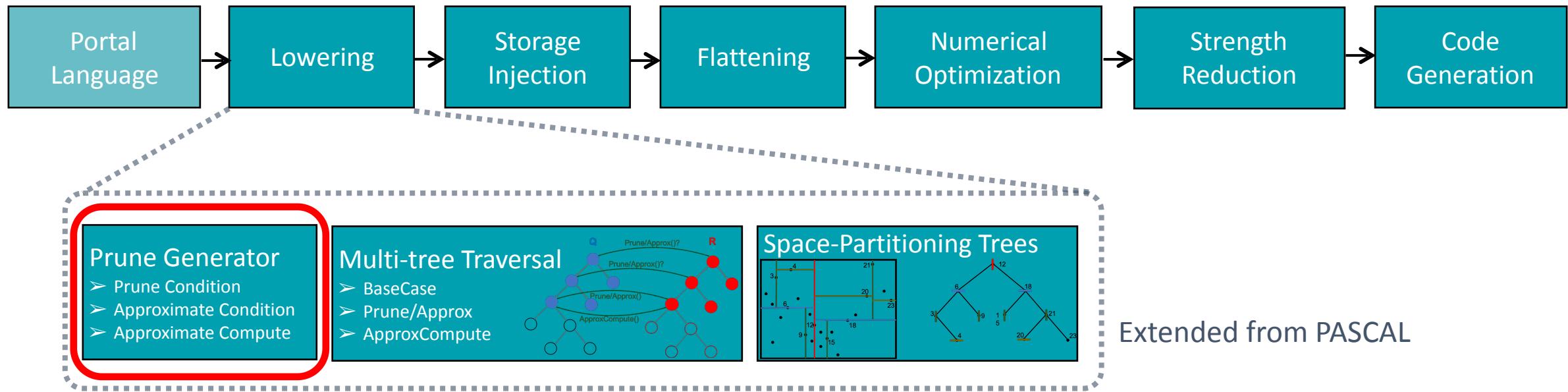


# Portal





# Portal





# Prune/Approximate Generator

---

- Nearest Neighbor (Pruning class)

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$



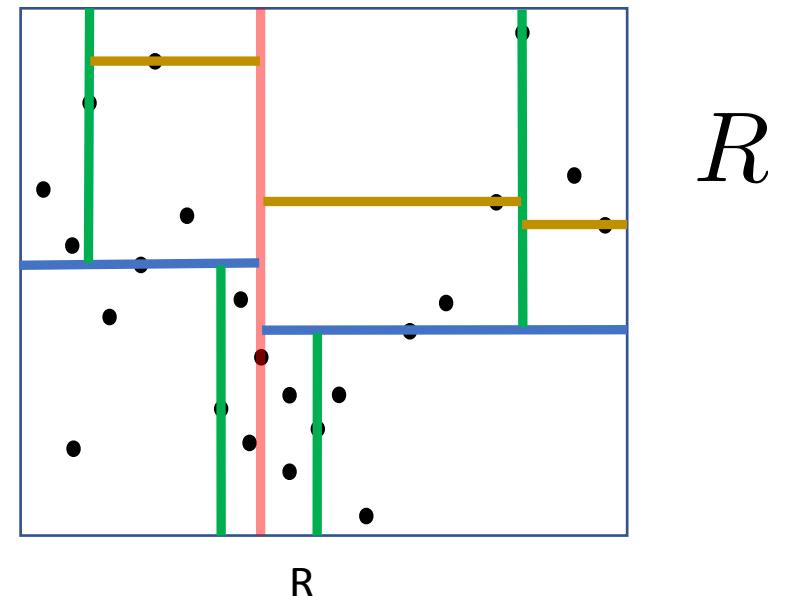
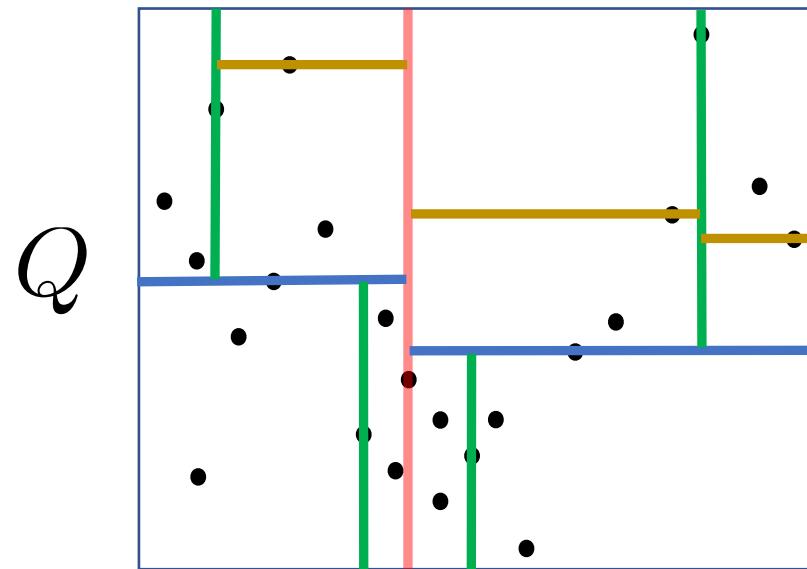
R



# Prune/Approximate Generator

- Nearest Neighbor (Pruning class)

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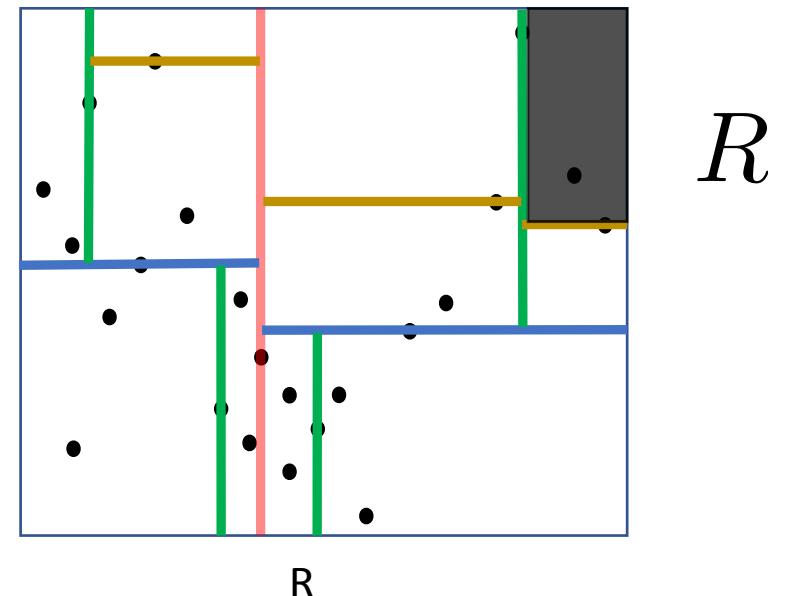
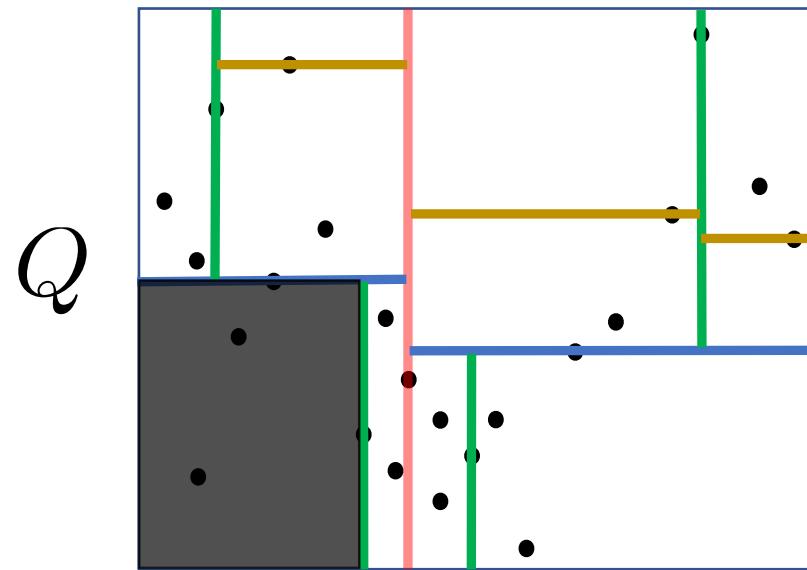




# Prune/Approximate Generator

- Nearest Neighbor (Pruning class)

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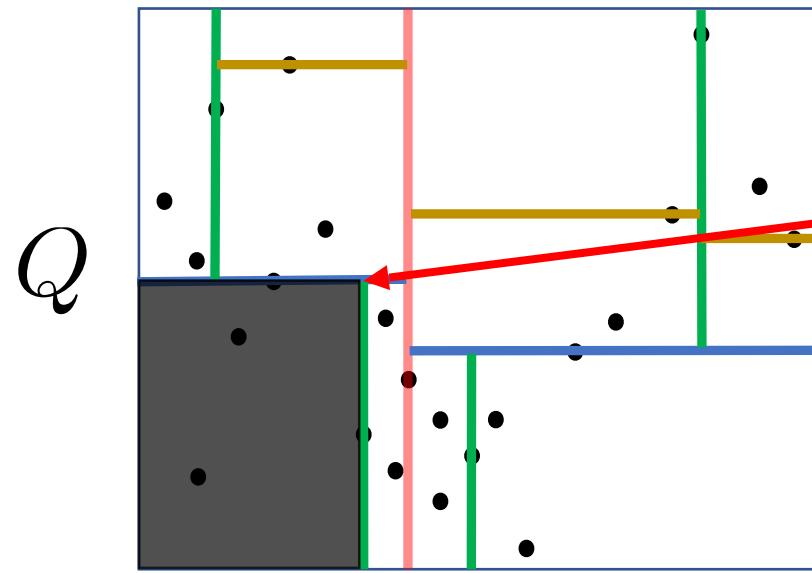




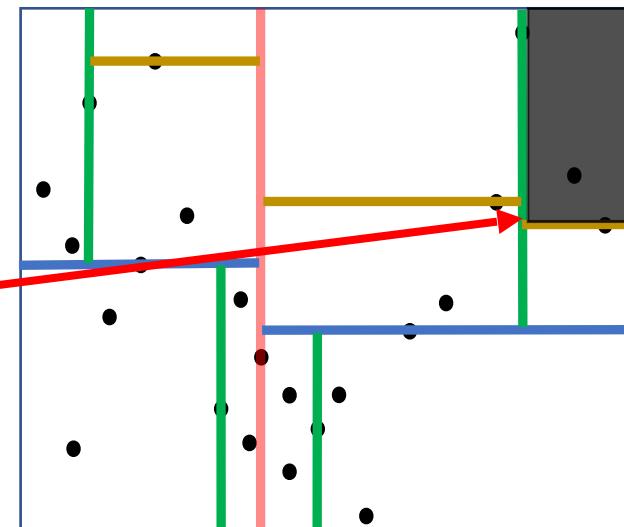
# Prune/Approximate Generator

- Nearest Neighbor (Pruning class)

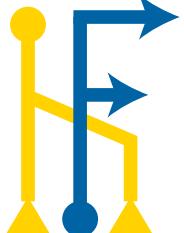
$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$



$d_{min}$



$R$





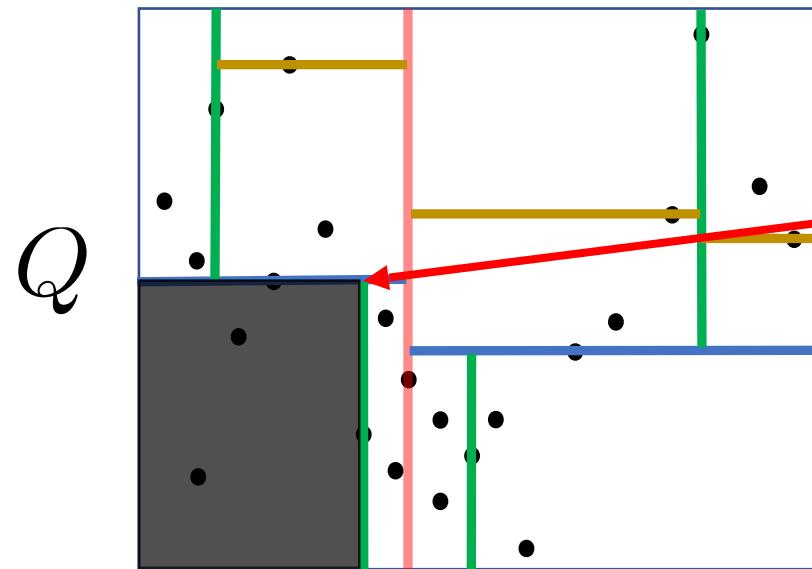
# Prune/Approximate Generator

- Nearest Neighbor (Pruning class)

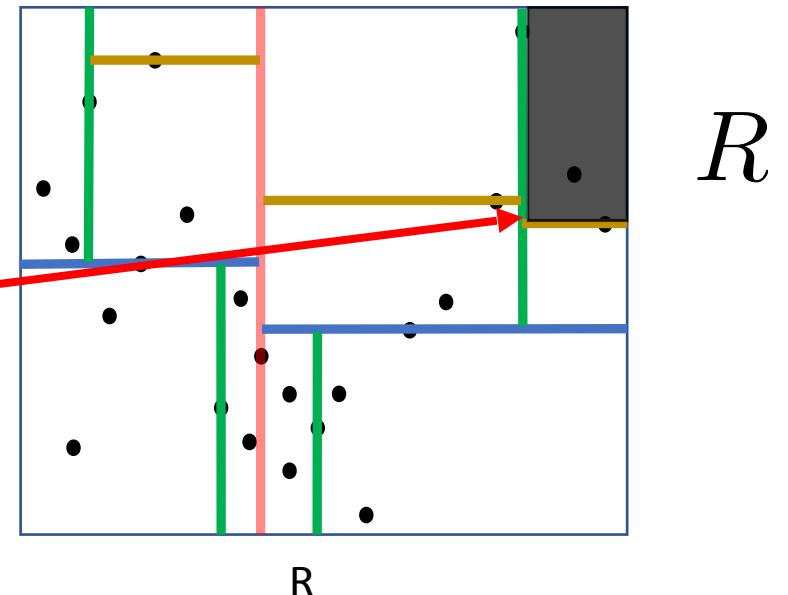
$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

- Prune condition

$$d_{sofar} < d_{min}$$



$$d_{min}$$



$R$





# Prune/Approximate Generator

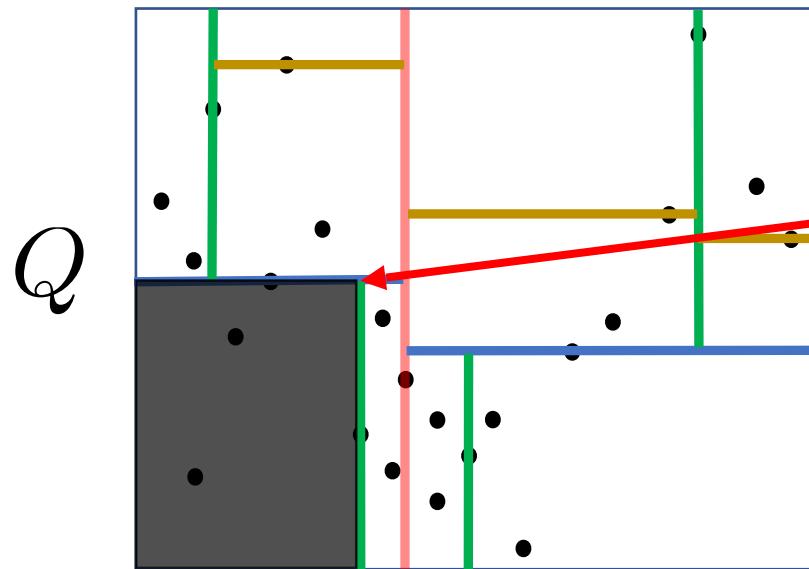
- Nearest Neighbor (Pruning class)

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

- Prune condition

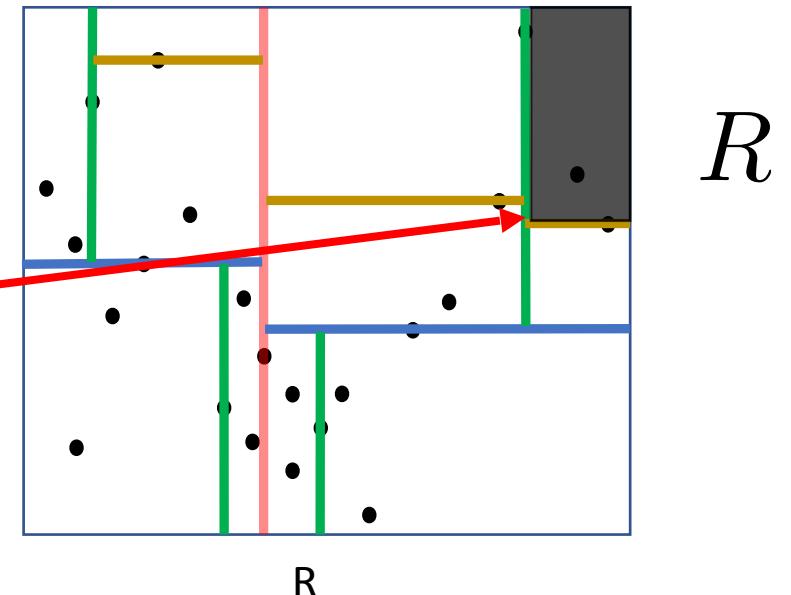
$$d_{sofar} < d_{min}$$

- Approximate compute



$d_{min}$

$Q$



$R$





# Prune/Approximate Generator

- Nearest Neighbor (Pruning class)

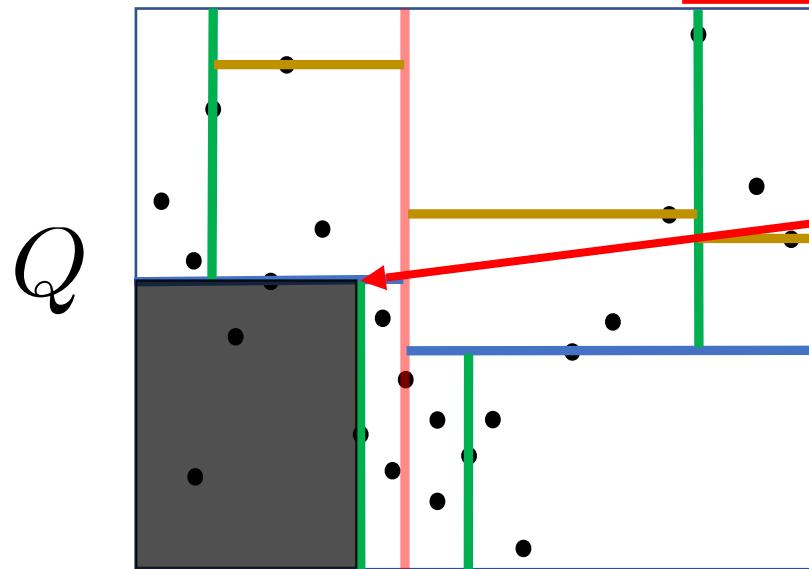
$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

- Prune condition

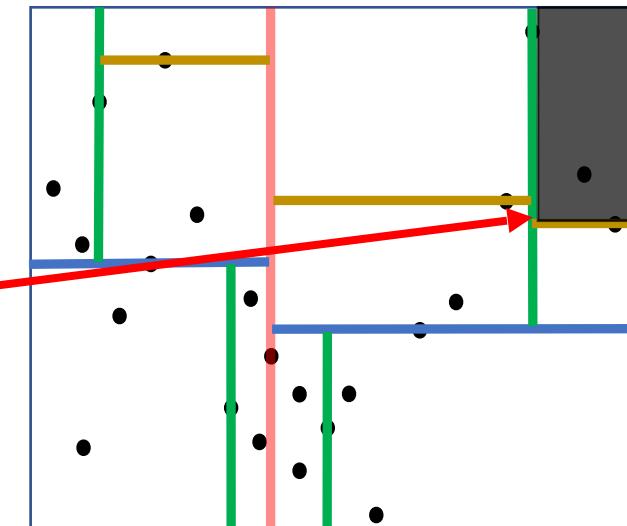
$d_{sofar} < d_{min}$

- Approximate compute

$return 0$



$d_{min}$



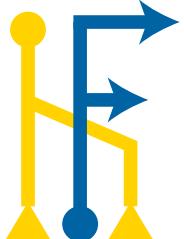
$R$





# Prune/Approximate Generator

---





# Prune/Approximate Generator

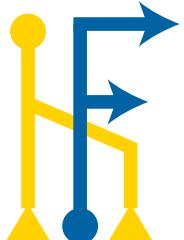
---

- Kernel Density Estimation (**Approximation class**)

- Approximate condition

$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$

- Approximate compute

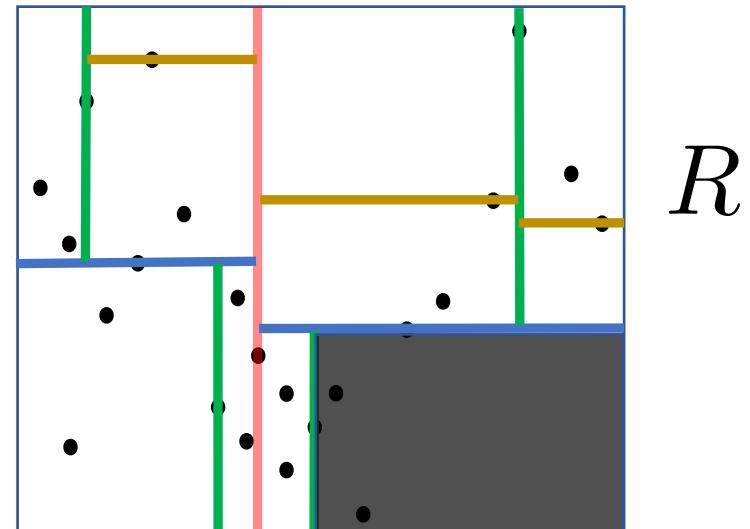
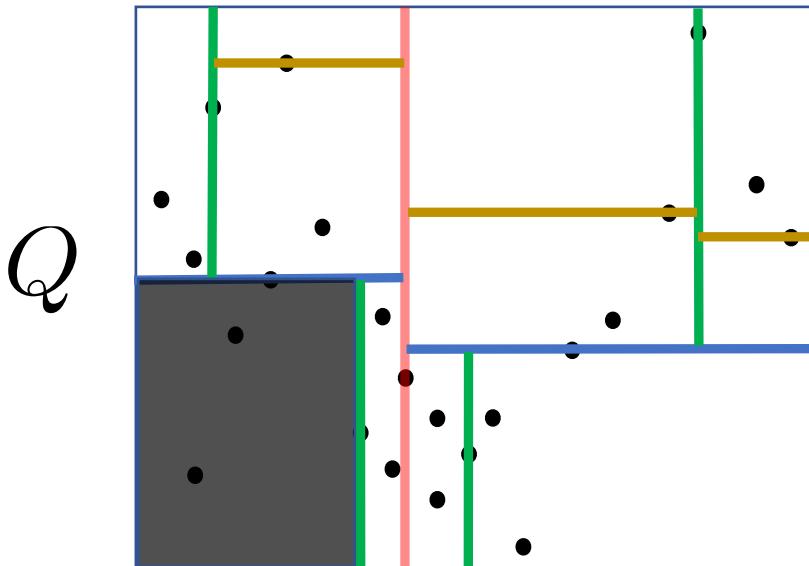




# Prune/Approximate Generator

- Kernel Density Estimation (**Approximation class**)
  - Approximate condition
  - Approximate compute

$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$





# Prune/Approximate Generator

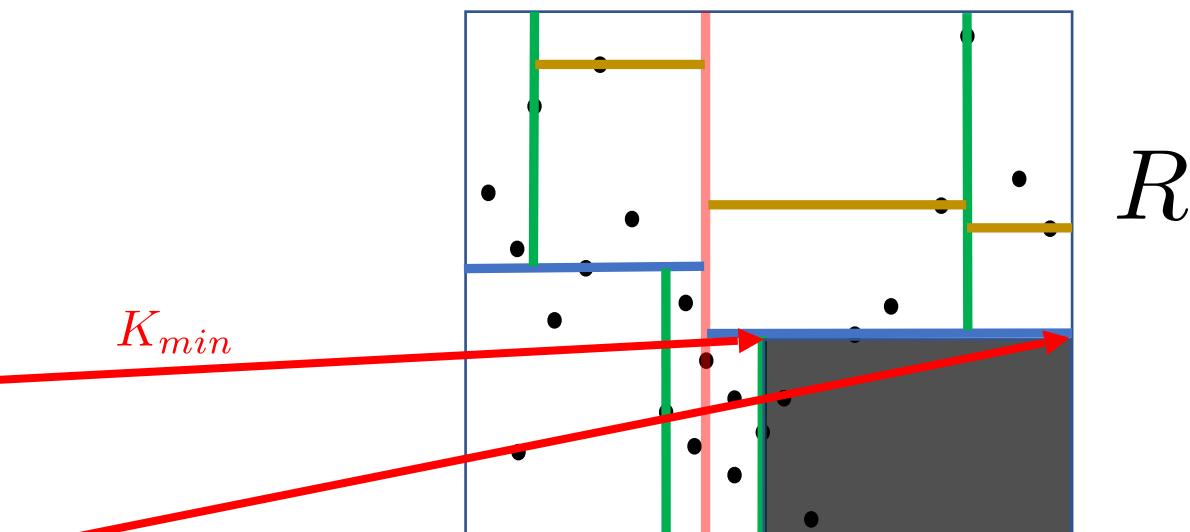
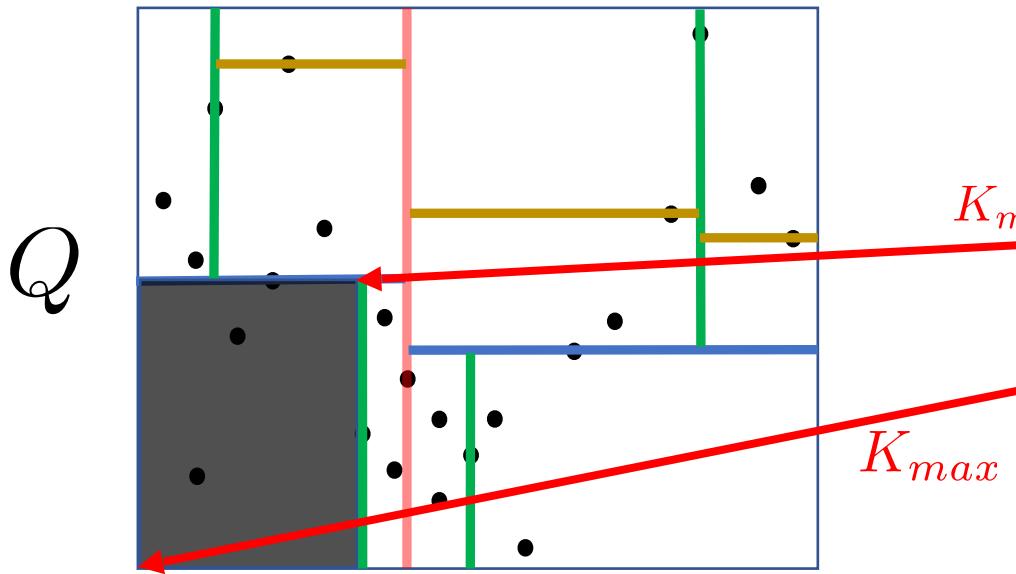
- Kernel Density Estimation (Approximation class)

- Approximate condition

$$K_{\max} - K_{\min} < \beta \times K_{center}$$

$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$

- Approximate compute





# Prune/Approximate Generator

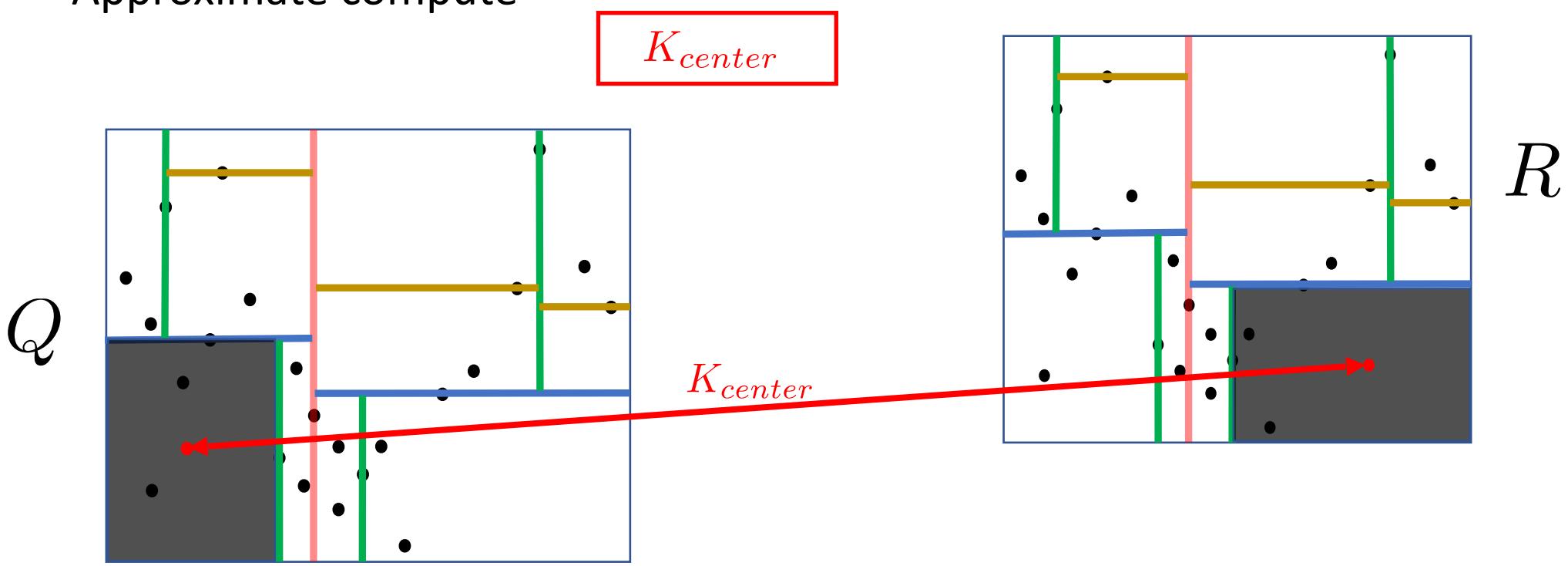
- Kernel Density Estimation (Approximation class)

- Approximate condition

$$K_{\max} - K_{\min} < \beta \times K_{center}$$

$$\forall q \in Q, \quad \sum_{r \in R} K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$

- Approximate compute





# Portal Compiler

---

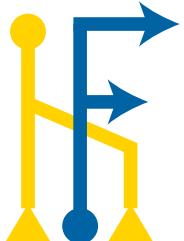




# Portal Compiler

---

Nearest Neighbor





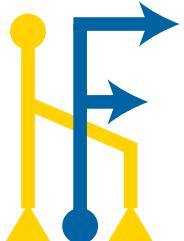
# Portal Compiler

---

Nearest Neighbor

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$





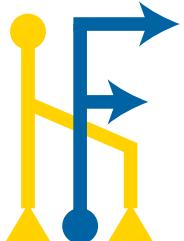
# Portal Compiler

Nearest Neighbor

Portal  
Language

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$





# Portal Compiler

Nearest Neighbor

Portal  
Language

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

Portal Language

```
Storage query("query_file.csv");
Storage reference("reference_file.csv");
Var q,r;
Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL, q, query);
expr.addLayer(PortalOp::ARGMIN, r, reference,
EuclidDist);
expr.execute();
Storage output = expr.getOutput();
```





# Portal Compiler

Nearest Neighbor

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$



Portal Language

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# Portal Compiler

Nearest Neighbor

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

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Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL, q, query);
expr.addLayer(PortalOp::ARGMIN, r, reference,
EuclidDist);
expr.execute();
Storage output = expr.getOutput();
```



```
/* Storage injection for outer layer */
alloc storage0[q.size]
for q in query.start ... query.end
    /* Storage injection for inner layer */
    alloc storage1 = max numeric limit
    for r in reference.start ...reference.end
        alloc t = 0
        /* Lowering the kernel function */
        for d in 0 ... dim
            t += pow((load(q,d)-load(r,d)),2)
        t = sqrt(t)
        /* Lowering the min functionality */
        storage1 = storage1 < t ? storage1 : t
    storage0[q] = storage1
```

```
/* Prune/Approximate condition for the two tree nodes
N1 (from query) and N2 (from reference) */
for d in 0 ... dim
    t += pow((load(N1min,d)-load(N2min,d)),2)
t = sqrt(t)
return (t > current_min_distance)
```

```
/* Nearest Neighbor is a pruning problem, hence there
is no approximation */
return 0;
```





# Portal Compiler

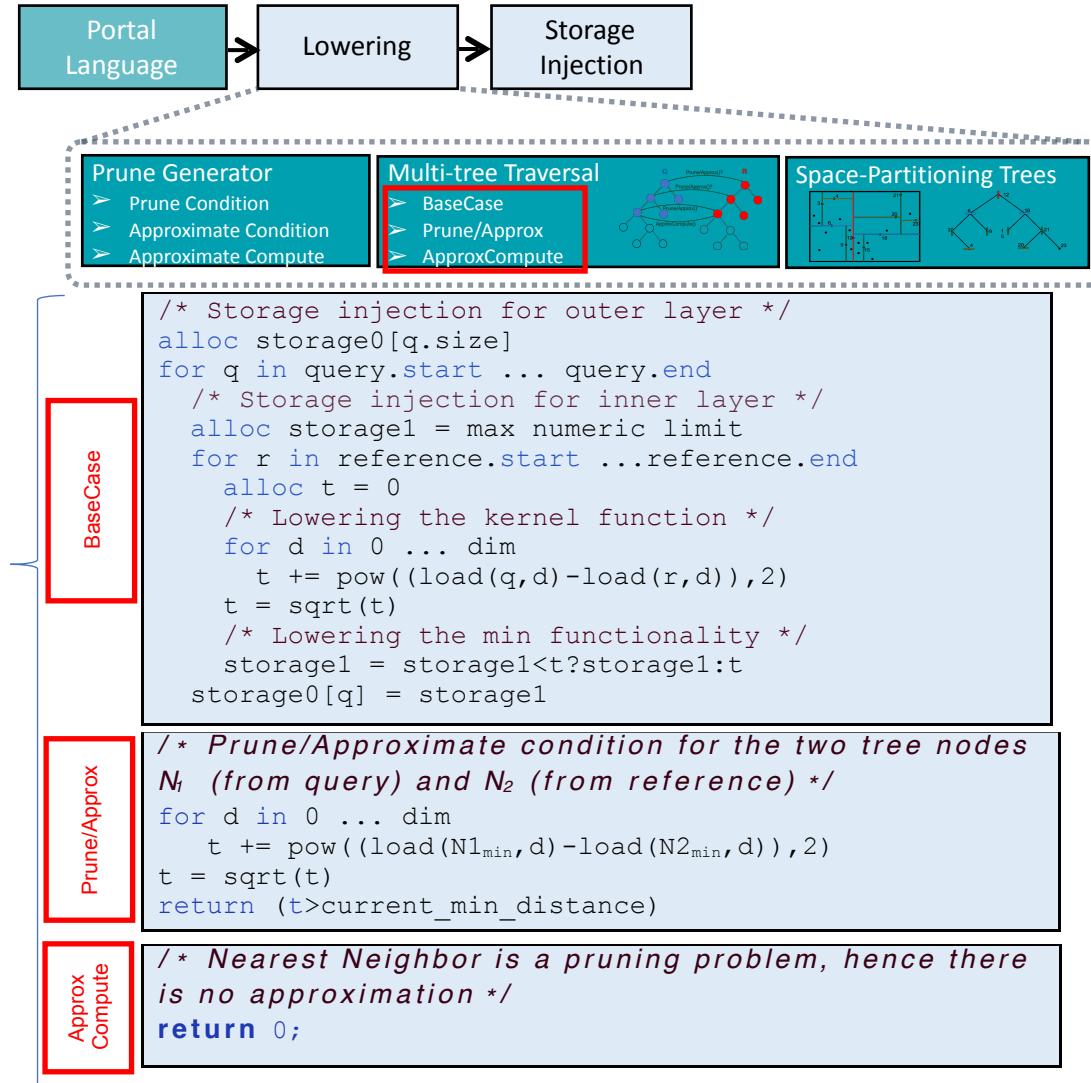
## Nearest Neighbor

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

Portal Language

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Storage reference("reference_file.csv");
Var q,r;
Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL, q, query);
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```





# Portal Compiler

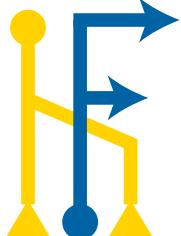
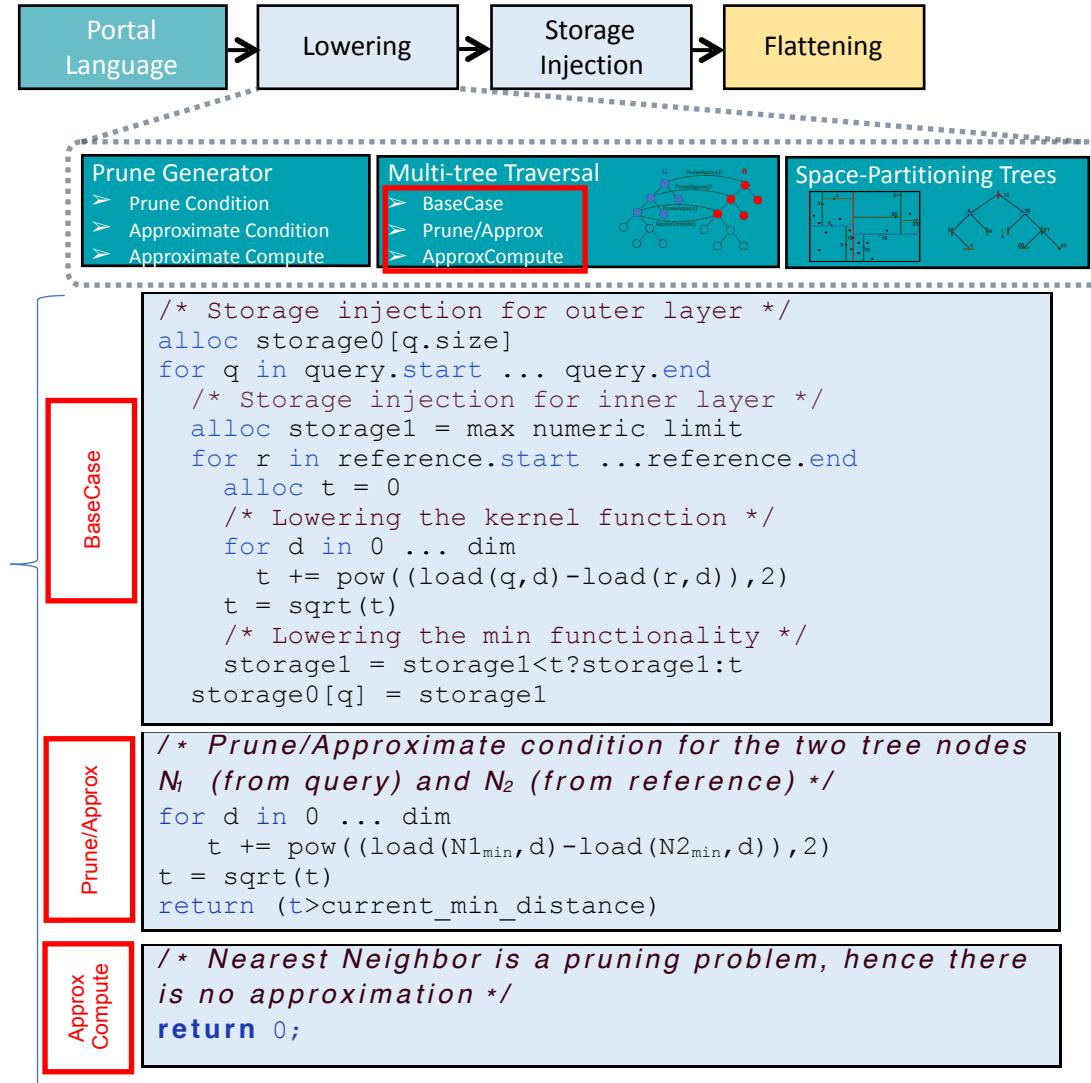
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# Portal Compiler

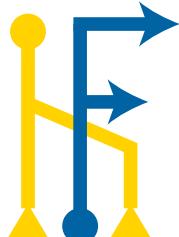
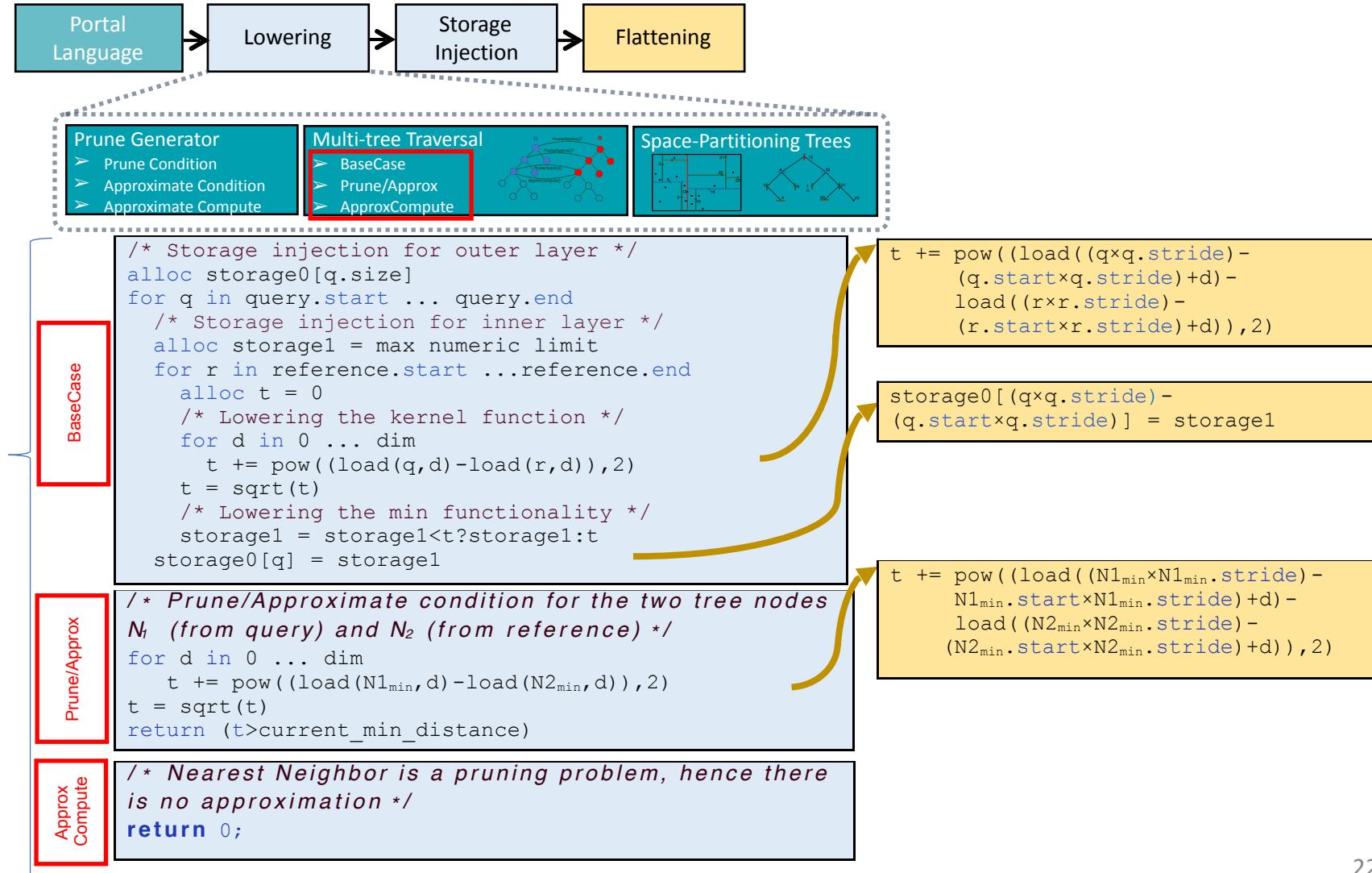
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expr.execute();
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```





# Portal Compiler

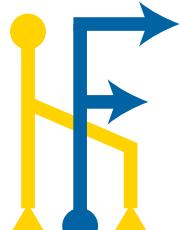
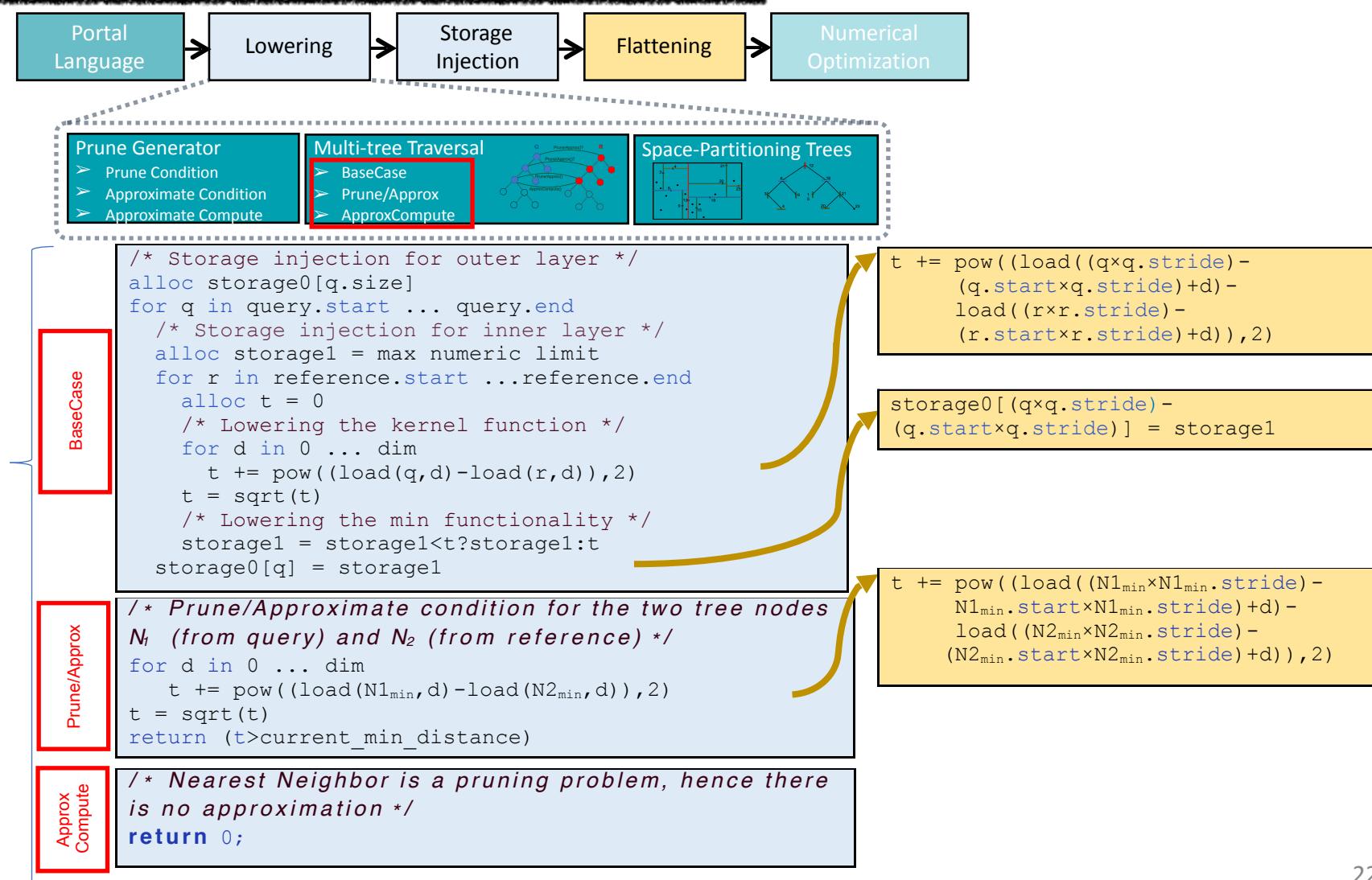
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Mathematical Equation

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# Portal Compiler

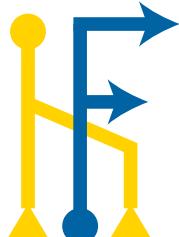
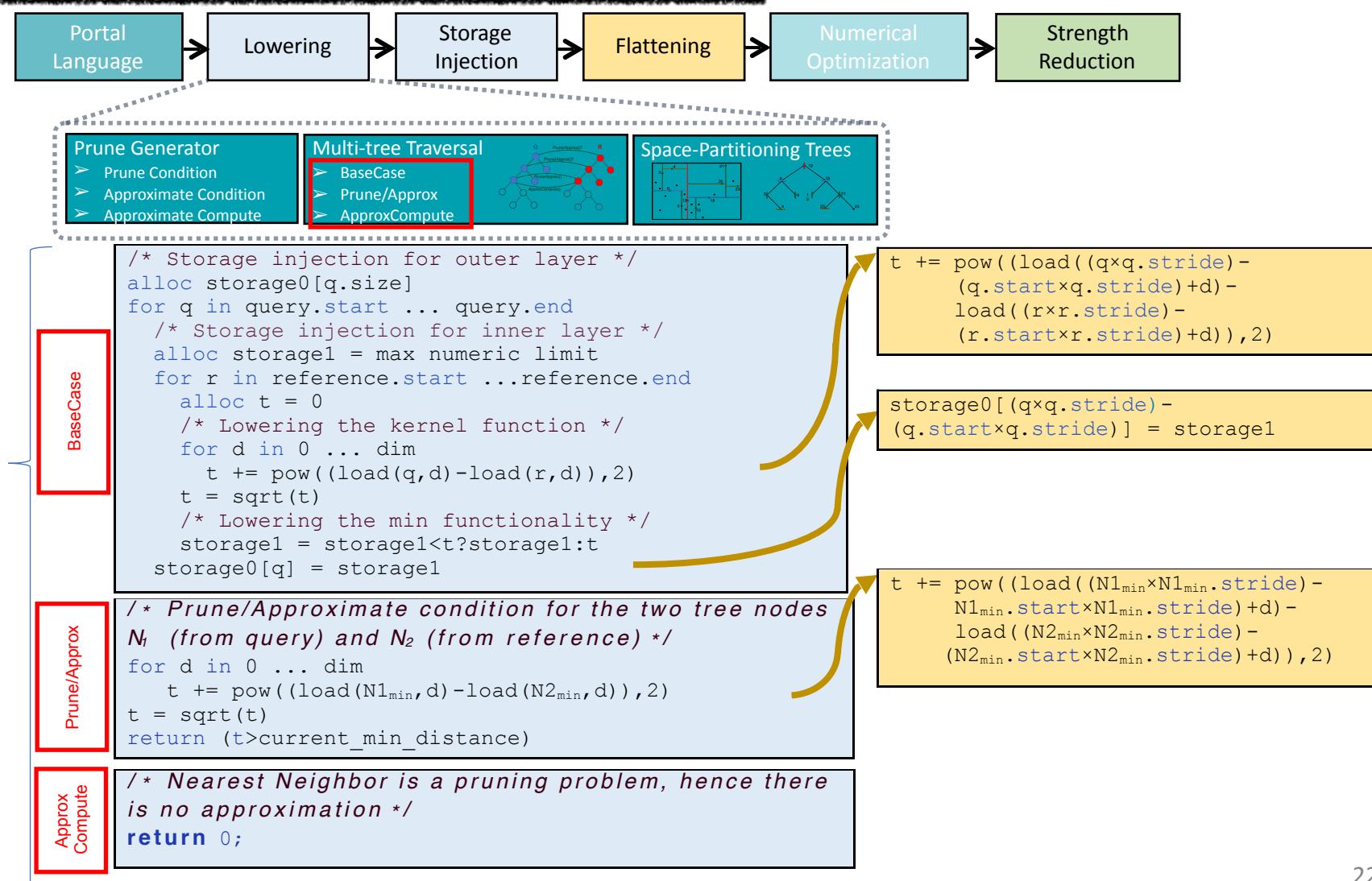
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Portal Language

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expr.execute();
Storage output = expr.getOutput();
```





# Portal Compiler

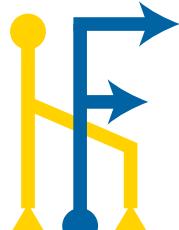
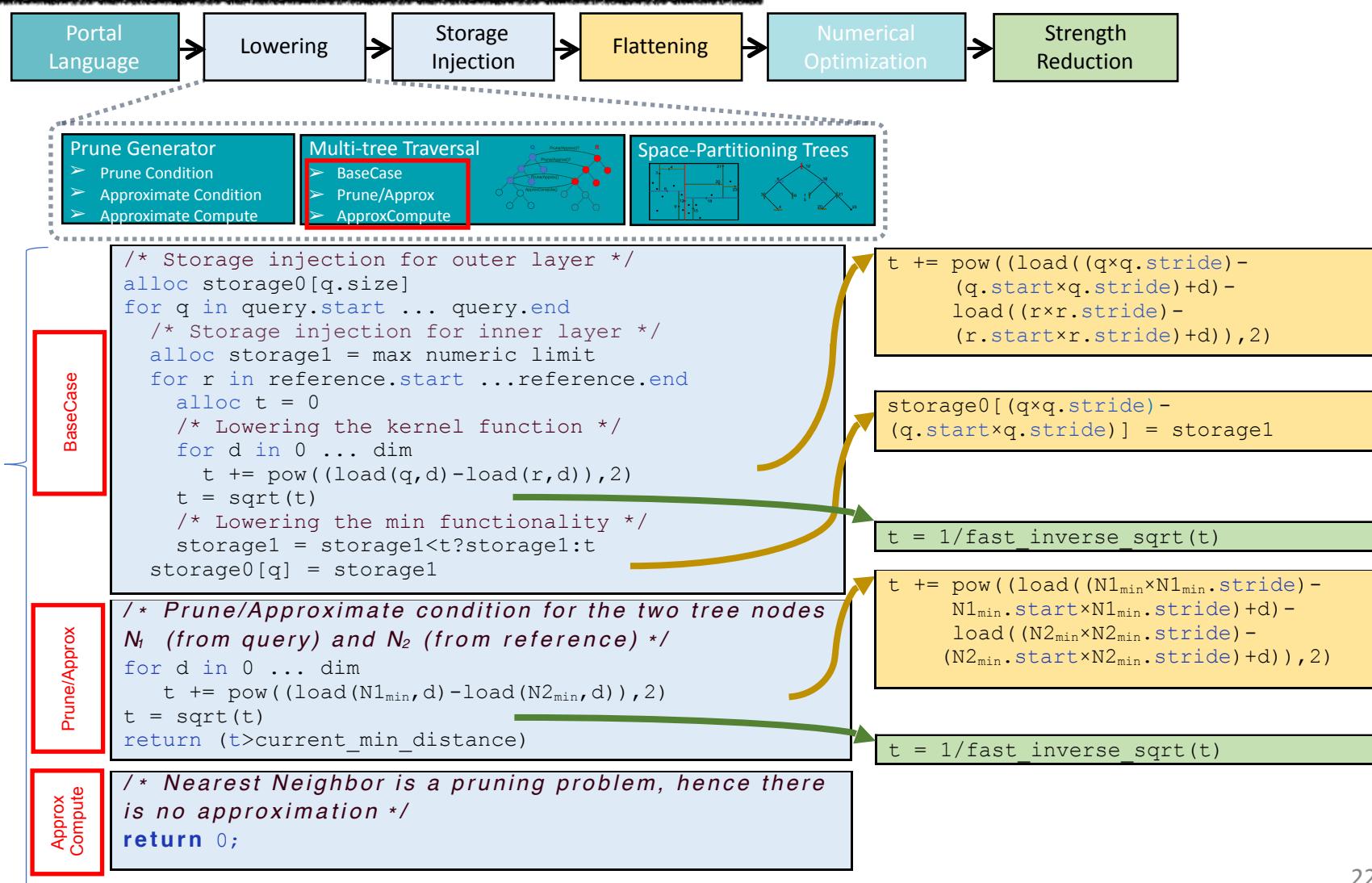
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Mathematical Equation

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Portal Language

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Var q,r;
Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
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expr.addLayer(PortalOp::ARGMIN, r, reference,
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Storage output = expr.getOutput();
```





# Portal Compiler

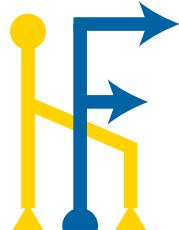
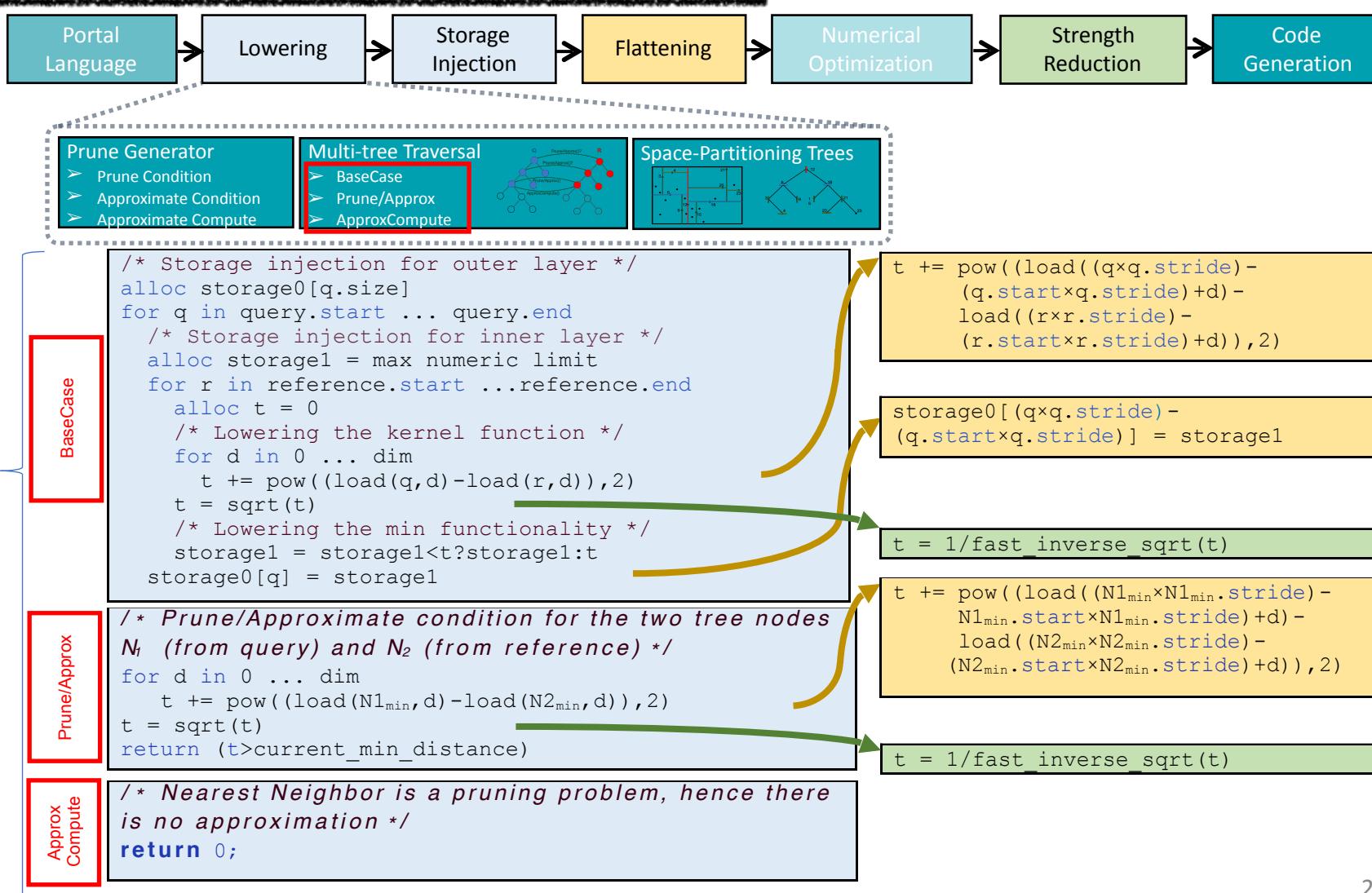
## Nearest Neighbor

Mathematical Equation

$$\forall q \in Q, \arg \min_{r \in R} \|x_q - x_r\|$$

Portal Language

```
Storage query("query_file.csv");
Storage reference("reference_file.csv");
Var q,r;
Expr EuclidDist = sqrt(pow((q-r), 2));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL, q, query);
expr.addLayer(PortalOp::ARGMIN, r, reference,
EuclidDist);
expr.execute();
Storage output = expr.getOutput();
```





# Portal Compiler

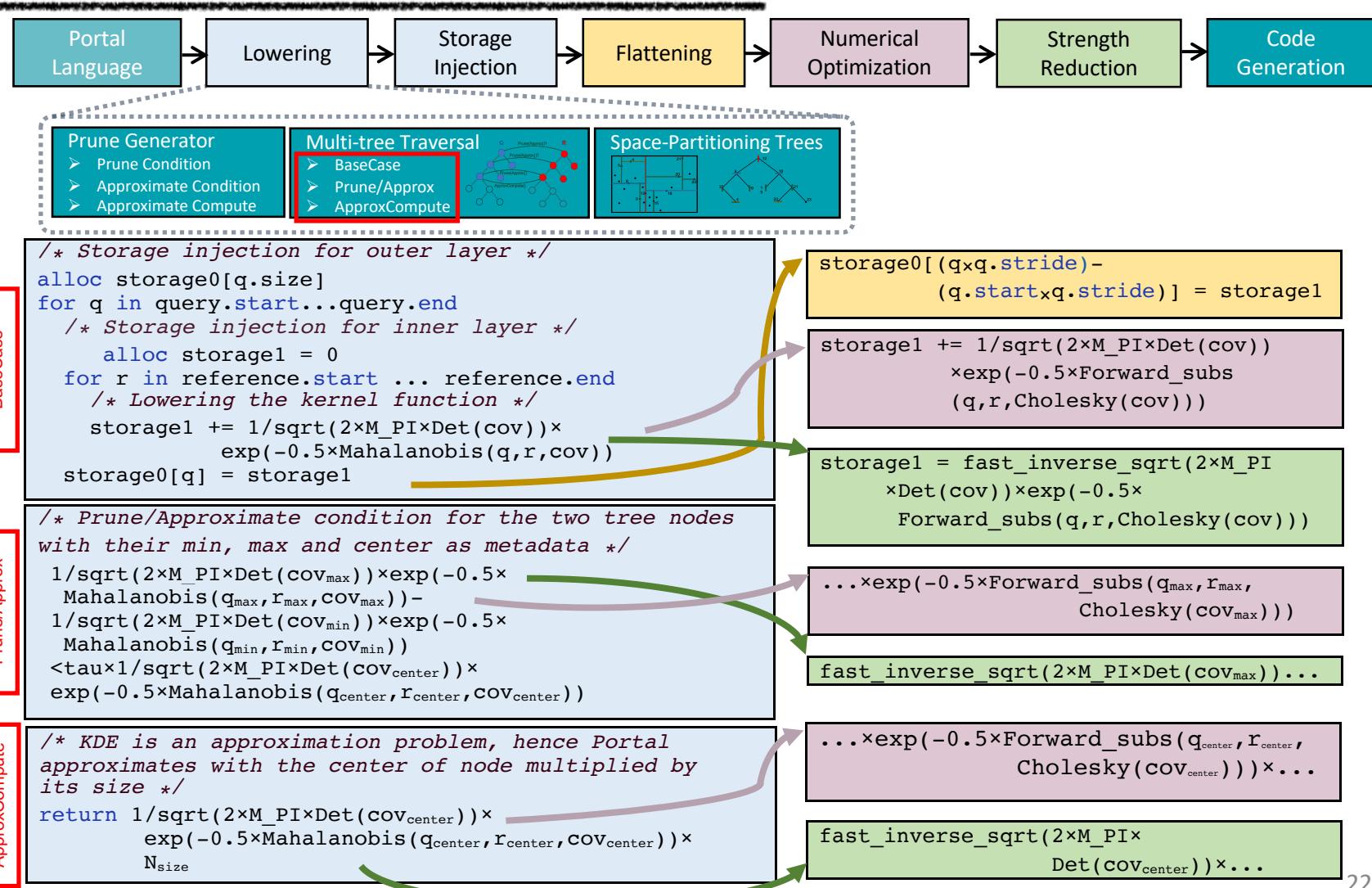
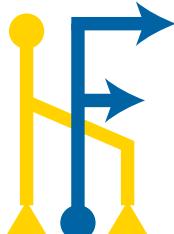
- Kernel Density Estimation

Mathematical Equation

$$\forall q \in Q, \quad \sum_r K_\sigma \left( \frac{\|x_q - x_r\|}{\sigma} \right)$$

Portal Language

```
Storage query("query_file.csv");
Storage reference("ref_file.csv");
Var q,r,cov;
Expr Kernel = 1/sqrt(2*M_PI*Det(cov))
  ×exp(-0.5×Mahalanobis(q,r,cov));
PortalExpr expr;
expr.addLayer(PortalOp::FORALL,q,query);
expr.addLayer(PortalOp::SUM,r,
reference,Kernel);
expr.execute();
Storage output = expr.getOutput();
```

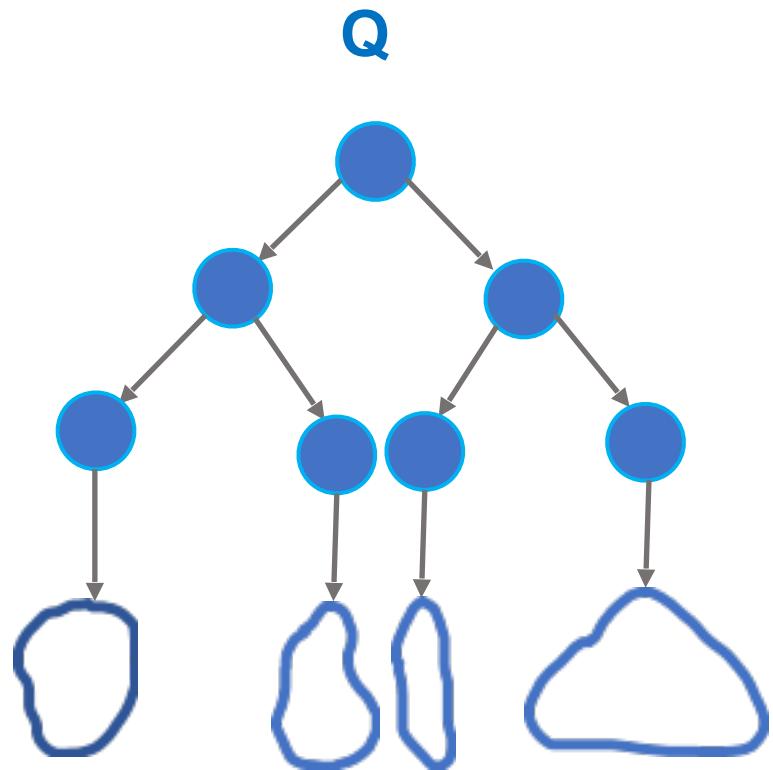




# Portal Compiler

---

- Code Generation
  - Using LLVM for X86
  - Parallelization on multi-tree traversal
    - Using OpenMP

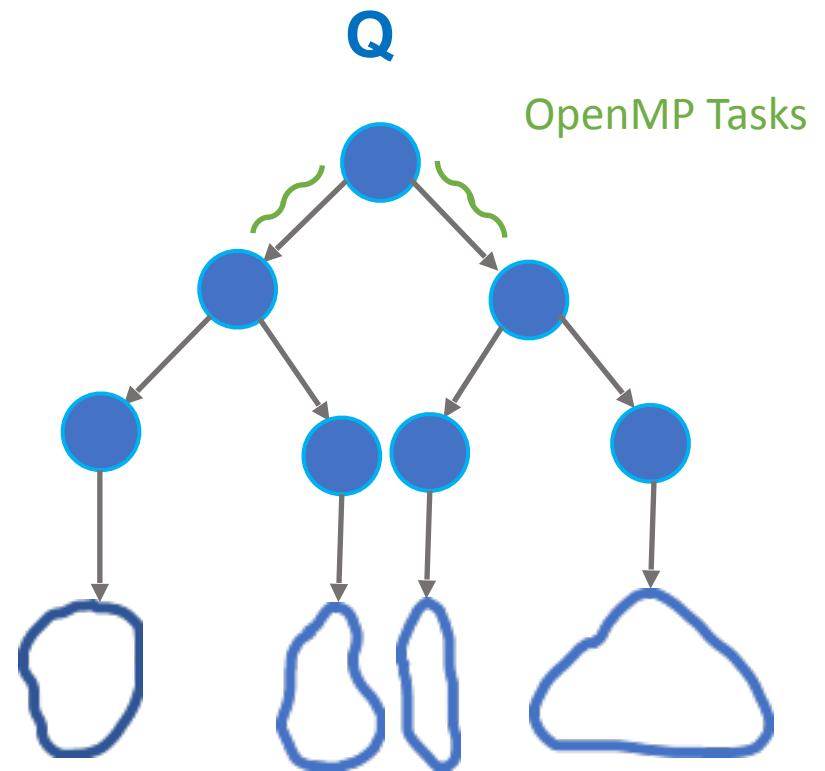




# Portal Compiler

---

- Code Generation
  - Using LLVM for X86
  - Parallelization on multi-tree traversal
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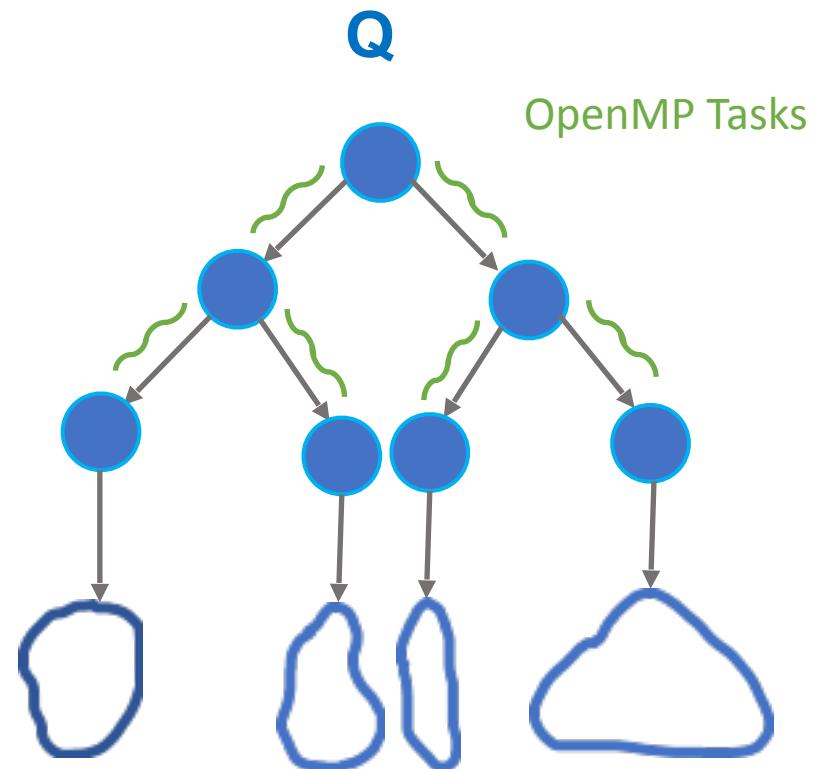




# Portal Compiler

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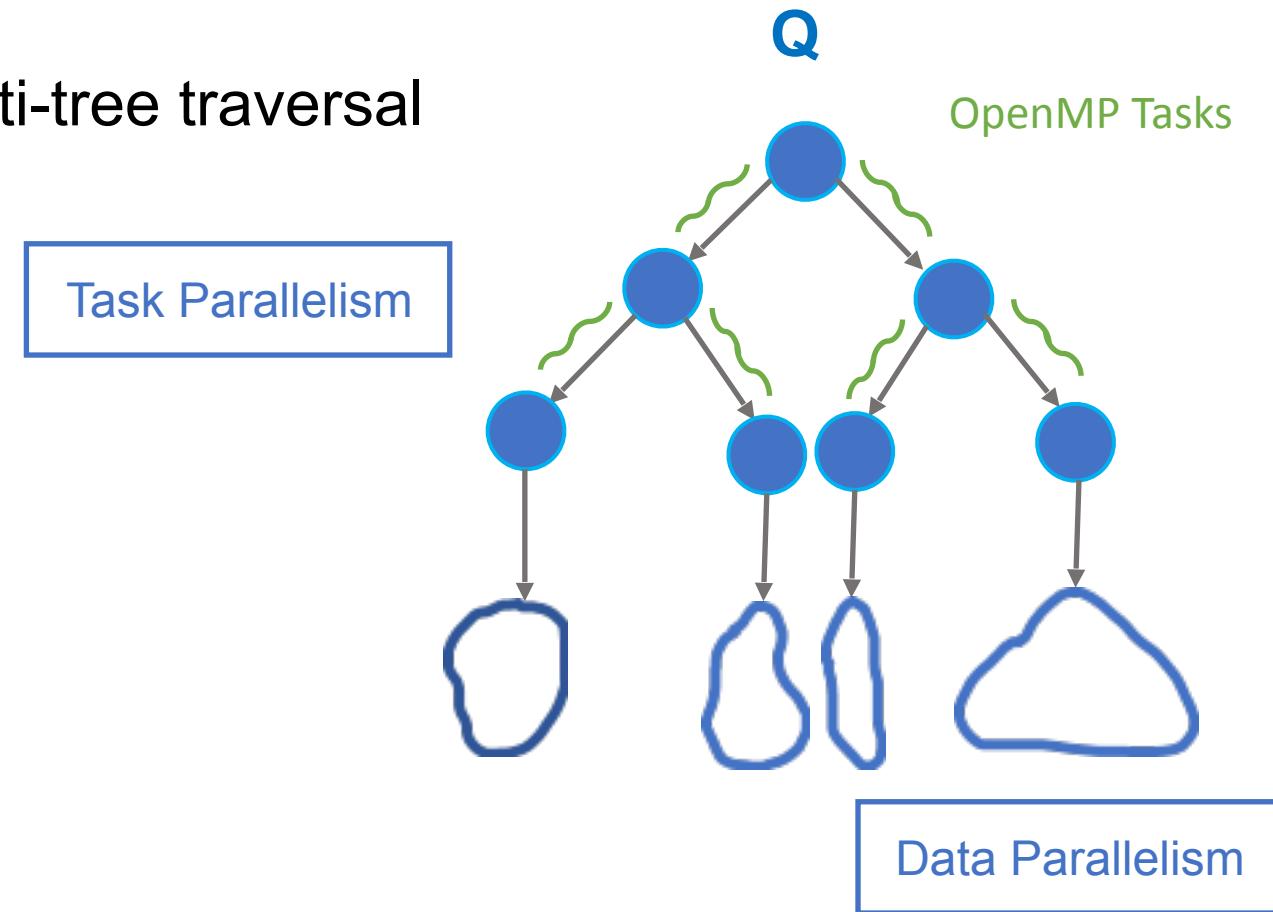
- Code Generation
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    - Using OpenMP





# Portal Compiler

- Code Generation
  - Using LLVM for X86
  - Parallelization on multi-tree traversal
    - Using OpenMP





# Experimental Setup

---

- Architecture and compiler
  - Dual-socket AMD EPYC 7551, each socket 64 cores
  - Clang compiler and LLVM version 6.0.0
  - Python v3.7.0 for scikit-learn v0.20.0 and MLPACK 3.0.3
- Benchmarks

Dataset	N	d
Yahoo!	41904293	11
IHEPC	2075259	9
HIGGS	11000000	28
Census	2458285	68
KDD	4898431	42
Elliptical	10000000	3



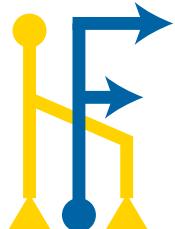


# Case Studies

Portal Performance Test (6 examples)

Problem	Operators	Kernel function
All Nearest Neighbor	$\forall, \arg \min$	$\ x_q - x_r\ $
All Range Search	$\forall, \cup \arg$	$I(h_{\min} < \ x_q - x_r\  < h_{\max})$
All Range Count	$\forall, \Sigma$	$I(h_{\min} < \ x_q - x_r\  < h_{\max})$
Naïve Bayes Classifier	$\forall, \arg \max$	$(1/\sqrt{2\pi \Sigma_k })e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)} P(C_k)$
Mixture Model E-step	$\forall, \forall$	$(1/\sqrt{2\pi \Sigma_k })e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)}$
K-means E-step	$\forall, \arg \min$	$\ x_q - x_r\ $
Mixture Model Log-likelihood	$\sum, \log \sum$	$(1/\sqrt{2\pi \Sigma_k })e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)}$
Kernel Density Estimation	$\forall, \Sigma$	$\phi(\frac{\ x_q - x_r\ }{h})$
Kernel Density Bayes Classifier	$\forall, \arg \max \Sigma$	$\phi(\frac{\ x_q - x_r\ }{h})P(C_k)$
2-Point Correlation	$\Sigma, \Sigma$	$I(\ x_q - x_r\  < h)$
Nadaraya-Watson Regression	$\forall, \Sigma$	$y_r \phi(\frac{\ x_q - x_r\ }{h})$
Thermodynamic Average	$\Sigma, \Sigma$	$\phi(\ x_q - x_r\ )$
Largest-Span Set	max, ..., max	$\Sigma(\ x_q - x_r\ )$
Closest Pair	max, ..., max	$\ x_q - x_r\ $
Minimum Spanning Tree	$\forall, \arg \min$	$\ x_q - x_r\ $
Gravitation Interaction (Barnes-Hut)	$\forall, \Sigma$	$\frac{\alpha_q \alpha_r}{\ x_q - x_r\ }$
Average Density	$\Sigma, \Sigma$	$I(\ x_q - x_r\  < h)$
Wave Function	$\forall, \Pi$	$\phi(\ x_q - x_r\ )$
Hausdorff Distance	max, min	$\ x_q - x_r\ $
Intrinsic (Fractional) Dimension	$\Sigma, \Sigma$	$I(\ x_q - x_r\  < h)$

Portal Validation Test (3 examples)





# Portal Performance

	Census			Yahoo!			IHEPC			HIGGS			KDD			LOC		
	Expert	Portal	% Diff	Expert	Portal	X shorter												
K-NN	22.8	23.9	4	84.6	85.2	2	8.7	9.1	4	186	191	3	21.4	22.6	5	867	13	67
KDE	1087	1129	3	133.7	139.5	4	39.2	41.7	6	411.9	430.9	4	926.5	949	2	626	15	42
RS	42.2	44.4	5	214.5	223.1	4	15	16.1	7	130.1	130.1	6	20.1	21.1	4	673	13	52
MST	374.1	391.7	4	918.4	946.1	3	200.8	211	5	478.3	486.2	2	273.6	281	3	956	54	17
EM	76.3	82.6	8	224.5	242.8	8	78.6	85.3	8	198.8	216.7	9	32.4	35.3	8	1681	104	16
HD	40.9	43.1	5	122.8	129.7	5	38.4	40.1	4	236.6	243.8	3	36.2	38.3	5	689	13	53

K-NN: K-Nearest Neighbor  
MST: Minimum Spanning Tree

KDE: Kernel Density Estimation.  
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RS: Range Search  
HD: Hausdorff distance

Times in second





# Portal Performance

Datasets

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# Portal Performance

	Datasets												Line-Of-Code					
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	Expert	Portal	% Diff	Expert	Portal	% Diff	Expert	Portal	% Diff	Expert	Portal	% Diff	Expert	Portal	% Diff	Expert	Portal	X shorter
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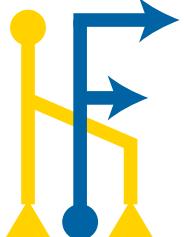
RS: Range Search  
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Times in second



# Portal Validation

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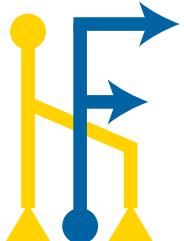




# Portal Validation

---

- Compare against State-of-the-art (S-O-A)





# Portal Validation

---

- Compare against State-of-the-art (S-O-A)
  - 2-point correlation (2-PC): MLPACK [JMLR 2014]





# Portal Validation

---

- Compare against State-of-the-art (S-O-A)
  - 2-point correlation ([2-PC](#)): MLPACK [[JMLR 2014](#)]
  - Naïve Bayes Classifier ([NBC](#)): Scikit-learn [[JMLR 2011](#)]

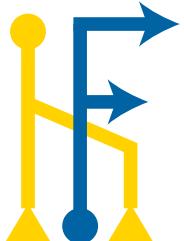




# Portal Validation

---

- Compare against State-of-the-art (S-O-A)
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# Portal Validation

- Compare against State-of-the-art (S-O-A)
  - 2-point correlation (2-PC): MLPACK [JMLR 2014]
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  - Barnes-Hut (BH): FDPS[PASJ 2016]

	Census			Yahoo!			IHEPC			HIGGS			KDD			Elliptical		
	S-O-A	Porta l	Spee dup	S-O-A	Portal	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Portal	Spee dup
2-PC	3529	53	66	37043	250	148	4281	26	162	17823	151	117	5134	31	165	5412	94	57
NBC	1337	87	15	3629	198	18	1699	88	19	5231	261	20	981	47	21	1026	194	5
BH	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	473	278	1.7

Times in second





# Portal Validation

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	Census			Yahoo!			IHEPC			HIGGS			KDD			Elliptical		
	S-O-A	Porta l	Spee dup	S-O-A	Portal	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Porta l	Spee dup	S-O-A	Portal	Spee dup
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Times in second

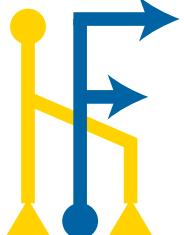




# Summary

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- Portal
  - Portal Language
    - Inspired by mathematical formulation
    - Chaining layers
    - Reduces line of code
  - Portal Compiler
    - Provides asymptotically optimal tree-based algorithms →  $O(N \log N)$  &  $O(N)$
    - Applies domain-specific optimizations
    - Easily extensible
    - Similar performance to hand-tuned expert code
- Portal Validation
  - Validated on a variety of problems
- Portal is open-source! [<https://gitlab.com/Nbody-Portal/Code>]





# Thank you!

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# Portal Compiler

---

- Numerical Optimizations
  - Mahalanobis distance

$$\mathcal{N}(x_i|\theta_k) = \frac{1}{\sqrt{2\pi |\Sigma_k|}} e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)}$$

- Cholesky decomposition(  $\Sigma = LL^T$  )

$$(x_q - \mu_r)^T \Sigma^{-1} (x_q - \mu_r) = (x_q - \mu_r)^T (LL^T)^{-1} (x_q - \mu_r)$$

- Forward substitution

$$Y = x_i - \mu_j$$

$$Y^T (LL^T)^{-1} Y = (L^{-1}Y)^T L^{-1} Y$$





# Why a Compiler?

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- There are hundreds of N-body problems and it's practically impossible to generate hand-optimized code for every single of them
- Hand-tuning is tedious and highly machine-specific
- Domain scientist expert in particular domain but not in parallel computing
- Scientist prefer to program in a high-level language which allows concise expression of their problem

